A Dynamic Programming Approach to Educational Choice: Theory, Econometrics, and Practice

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Abstract

Dynamic choice models, and in particular discrete choice dynamic programming (DDP) models, have become an increasingly popular way for social scientists to model individual behavior such as educational investment over the life cycle taking into account resource limitations, future consequences of current decisions and uncertainty about the future. But DDP models are difficult to estimate due to their substantial computational requirements and their seemingly extreme data requirements. In this paper we use simulated data as well as the National Longitudinal Survey of Youth (NLSY) to identify aspects of the DDP that can be relaxed in practice. We also examine how much information across time per individual is "enough" to estimate DDP parameters. Finally, we present an application of an extensive DDP model of educational investment using data from the NLSY and simulate several policy changes and their welfare implications.

Understanding people's investment in human capital including formal education has been a rich area of social science research for many years. While many theoretical models emphasize the dynamic nature of people's choices, most empirical work considers only static models. Static models, however, cannot answer the question of how a proposed change in the existing educational system might change people's decisions because these hypothetical changes take us outside the environment in which these models are estimated. This shortcoming makes static models a less desirable tool for addressing these types of policy questions.

Dynamic choice models, and particularly discrete choice dynamic programming (DDP) models provide a useful alternative for empirical research. Not only are these models consistent with the theory, but they are also valuable in describing individual behavior over the life-cycle. These models allow the analyst to account for an individual's resource limitations, imperfect information about his or her ability relative to others, uncertainty about future events, and future consequences of current decisions. DDP models have become increasingly popular and used in a wide range of applications including career choice, educational investment, fertility, and retirement.[1] Moreover, these models are particularly well suited for welfare analyses and policy evaluations, which are key components of any empirical economic analysis.

Despite the continuing improvement of computer hardware and software, DDP models are often difficult to use in practice. Depending on the complexity of the models analyzed, these models can be computationally intensive and hence quite expensive to estimate. For example, allowing individuals to have several choices each period, incorporating unobserved heterogeneity of individuals, or letting the uncertainty associated with each period be correlated across time can radically increase the complexity, and therefore the feasibility, of the actual estimation.

This paper advances this technology by identifying aspects of the DDP that can be relaxed in practice. First, we use simulations to estimate over-specified versions of a general DDP model of educational choice and several static models to investigate the consequences of misspecification. Second, we use the simulations to identify the minimum amount of information per individual required to estimate DDP parameters accurately. Finally, we apply what we've learned to the National Longitudinal Survey of Youth (NLSY) and estimate an extensive, yet tractable, model of educational choice. Despite our focus on educational choice, our methods and findings are applicable to a wide range of other research topics. The remainder of the discussion describes our analysis in more detail.

In order to gain a better understanding of the shortcomings associated with using a static model or an over-simplified DDP, we begin by specifying a very general model of

[1]See, for example, Buchinsky and Leslie (2002), Gustman and Steimeier (2003), Benítez-Buchinsky and Rust (2003), Miller (1984), Wolpin (1984), Keane and Wolpin (1997), and Stock and Wise (1990).

educational choice and use this to simulate a large number of individuals over several time periods to form a test data set. Our framework and notation borrows heavily from Keane and Wolpin (1997). In each period, an individual chooses between either work, school, or staying at home based on the potential current period reward, and the expected discounted future utility conditional on the current period choice:

$$V(S(a), a) = \max_{m} V_m(S(a), a)$$

$$V_m(S(a), a) = R_m(S(a), a) + \delta E[V(S(a+1), a+1|S(a), d_m(a)) = 1]$$

We denote the choice as m and let $m \in \{\text{work,school,home}\}$. Age is denoted as a and $d_m(a)$ is an indicator equal to 1 if the individual chooses activity m at age a. The state of the world for the individual at age a is written as $S(a)$ and includes years of education, years of experience, and the current and past history of random shocks.

Per period rewards for a choice m are written as $R_m(S(a), a)$. These rewards can be both pecuniary (e.g., wage if the activity is working) and non-pecuniary (e.g., the "utility" gained from choosing to stay at home). δ is the individual's discount factor. Much of the complexity of the model is embedded in the structure of the reward functions. Wages are a function of current education and experience and each reward includes a random shock which is unobserved but known to the individual during the period it occurs. Our general model allows individuals to be one of several unobserved types and lets the random shocks that occur each period be correlated across rewards and over time.

We show that the model's parameters can be consistently and practically estimated from our simulated data set and then estimate over-specified versions of the model. In particular, we estimate models that assume no unobserved heterogeneity and see how our parameter estimates change both in value and in precision. We also estimate the parameters of a model that assumes no correlation of random shocks across time. For the sake of comparison, we estimate a variety of static models in order to explore the consequences of this type of misspecification. We then evaluate and compare the quality of the policy recommendations that are derived from these different DDP and static models.

Next, we identify how much information across time per individual is "enough" to estimate the parameters of a DDP. Most DDP models have seemingly extreme data requirements, which seriously limit the type of data that can be used to estimate these models. Most empirical work exploits data sets with at least ten observations across time per individual and few data sets (e.g., the National Longitudinal Surveys (NLS) including the NLSY and the Panel Study of Income Dynamics (PSID)) can support this without relying on individuals' retrospective reports. We estimate our general model using various subsets of the simulated data and the NLSY data slowly dropping different observation years until we can no longer practically estimate the model parameters. Depending on assumptions about correlation of shocks across time, we find that it is sometimes feasible to estimate DDP's using panel data sets containing as few as 3 observations per individual over the course of several years. This finding suggests that data requirements for this modeling strategy are not as extreme as they may initially appear.

Finally, we apply our lessons learned to an analysis using the NLSY. We estimate an extensive DDP model of educational choice and evaluate the results in light of our earlier findings. We conduct several simulations of hypothetical policy changes and evaluate their welfare implications using our full model, several over-specified variants and a static model for comparison.

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