LONG RANGE TRENDS IN ADULT MORTALITY: MODELS AND PROJECTION METHODS

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Abstract
This study has two objectives: (1) to test a new model for the pattern of change over time in age-specific adult mortality rates, and (2) to develop a new methodology for projecting future trends in adult mortality. The first part of the paper presents a test of the goodness-of-fit of the logistic model for the force of mortality using data from the Human Mortality Data Bank for females and males aged 25-109 in 14 populations. The results from this exercise suggest a new version of the logistic model which is called the *shifting logistic model,* because the senescent mortality function is assumed to shift to higher ages over time. This approach contrasts with the conventional view that mortality declines as adult life expectancy rises. The last part of the paper proposes a new projection method based on the shifting mortality model. This method is compared with the widely used Lee-Carter procedure.

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Over the past two centuries life expectancy at birth in the industrialized ("developed") world approximately doubled, reaching 79 years for females and 72 years for males in 2000-2005 (United Nations, 2002). Much of this rise is attributable to large reductions in infant and child mortality. Mortality among the young is now so low, however, that further declines will have little impact on future trends in life expectancy. Future increases in life expectancy will therefore require additional reductions in adult mortality. This study will examine past trends in the age pattern of adult mortality and discuss their implications for long range mortality projections.

The description of observed age patterns of adult mortality with mathematical models is one of the oldest and most important topics in demography. The number and complexity of mortality models have grown rapidly since Gompertz proposed the first "law of mortality" in 1825. These developments have been made possible by the increasing availability of detailed mortality statistics in many countries. A good model provides a simple but adequate mathematical description of mortality by age and/or time. The objective is to identify fundamental and persistent patterns in the data and summarize them with as few parameters as possible. Models have found many uses, including smoothing of data, construction of model life tables, comparative analyses, testing of theories and forecasting (Keyfitz, 1984; Tabeau et. al., 2001).

A concise model description of past mortality trends provides the basis for projections. The theory and practice of forecasting mortality have evolved rapidly in recent decades and there are many ways to make forecasts (Keyfitz, 1991; Lee, 1998; Olshansky, 1988; Pollard, 1987; Tabeau et al., 2001). Projections for the short run typically rely on simple extrapolation of historical trends in mortality rates, in life expectancy or in model parameters. However, in projections for periods of more than a few decades, simple linear extrapolation can lead to implausible results and expert judgment is then often used to decide which long-range levels or trends are most probable. For example, experts may identify a target for life expectancy at birth in a future year. This has been the approach used by the United Nations, the World Bank and many national statistical agencies to make long-range population projections.

This study has two overall objectives: (1) to test a new model to describe the pattern of change over time in age-specific adult mortality rates, and (2) to develop a new methodology for projecting future trends in adult mortality. The first section of the paper presents a brief overview of models for the age pattern of adult mortality and a test of the goodness-of-fit of the logistic model for the force of mortality. This test uses data from the Human Mortality Data Bank for females and males aged 25-109 in 14 populations. The results of this exercise suggest a new version of the logistic model which will be called the *shifting logistic model,* because the adult mortality function is assumed to shift to higher ages over time. The second part of the paper applies this new model to decompose the rate of change in the force of mortality schedule into "background" and "senescent" components. This analysis provides crucial insights into the assumptions made by projection methods that rely on extrapolation of the rate of mortality improvement. The final section proposes a new projection methodology based on the

shifting mortality model. The new method is compared with the Lee-Carter procedure which is one of the most widely used methods for projecting mortality.

I. Models for the force of mortality

a) Age pattern

Mortality rates in a wide range of populations show an approximately exponential rise with age for adults. A simple parametric model proposed by Gompertz (1825) summarizes this pattern:

$$\mu(x) = \alpha e^{\beta x}, \qquad (1)$$

where $\mu(x)$ denotes the force of mortality at age x. The two parameters α and β are positive; α varies with the level of mortality and β measures the rate of increase in mortality with age.

For many purposes the Gompertz model provides a satisfactory fit to adult mortality rates. However, close inspection of the difference between model estimates and observed death rates often reveals systematic underestimation of actual mortality at youngest adult ages (under 40) and overestimation at the oldest ages (over 80). The deviation at lower ages is addressed by Makeham (1860) with the addition of a constant to the Gompertz model

$$\mu(x) = \alpha e^{\beta x} + \gamma \qquad (2)$$

The new parameter γ is usually referred to as background mortality which is the same for all ages. A detailed analysis of (2) is provided by Gavrilov and Gavrilova (1991).

The Makeham model represents a clear improvement over the Gompertz model at younger ages, but it still overestimates mortality at the oldest ages. This deviation can be addressed in a number of ways, most simply by the following logistic model (Thatcher, 1999, Thatcher et al.,1998) :

$$\mu(x) = \frac{\alpha e^{\beta x}}{1 + \alpha e^{\beta x}} + \gamma \qquad (3)$$

At lower adult ages the force of mortality estimated with models (3) and (2) are very similar, because the denominator of the first term in (3) is close to 1. At the oldest ages, however, the two models diverge as (3) levels off at $1+\gamma$ while (2) has no limit.

More complex logistic models with additional parameters have also been proposed (Beard, 1971; Horiuchi and Wilmoth, 1998; Perks, 1932; Thatcher et al., 1998). Based on a detailed comparison of different models Thatcher (1999) and Thatcher et al. (1998) recommend (3) because it provides an excellent fit to mortality rates over the entire adult age range with relatively few parameters.

The good fit of the logistic model (3) is demonstrated in Figure 1 which presents observed and estimated values of $\mu(x)$ for Swedish females in 1875, 1950 and 2000. The proportion of the variance explained by the model equals 0.9997 in 1875, 0.9996 in 1950 and 0.9985 in 2000.

To confirm these results for other populations the model given by (3) was fitted to annual mortality data from 1950 to 2000 for 14 countries, separately for males and females. All countries in the Human Mortality Databank outside Eastern European were included: Austria, Canada, Denmark, England and Wales, Finland, France, Italy, Japan, Netherlands, Norway, Sweden, Switzerland, USA, West Germany. Table 1 presents averages of annual estimates for α, β and γ for females and males in each of the 14 countries for all available years 1950-2000.[1] These results are discussed further below but for now it should be noted that the model fits very well in all these countries (see next to last column in Table 1). The fit is about the same for females (R^2 averages 0.9993 for the 14 countries) as for males (R^2 averages 0.9996).

Although the simple logistic model is well suited for present purposes, its fit is not perfect. An examination of differences between observed and fitted values reveals small systematic overestimation of mortality between ages 60 and 80, as well as some underestimation at the highest ages among females in a number of countries. This pattern is consistent with the findings of Himes et al. (1994).

In the following analysis of trends in adult mortality it is useful to distinguish between senescent mortality which rises with age and background mortality which does not vary with age (Gavrilov and Gavrilova, 1991, Horiuchi and Wilmoth, 1998, Makeham, 1860). The sum of these two components equals the force of mortality:

$$\mu(x,t) = \mu_s(x,t) + \mu_b(t) \qquad (4)$$

where $\mu_s(x,t)$ = senescent force of mortality

$\mu_b(t)$ = background force of mortality

For the logistic model the first term on the right hand side of (3) equals $\mu_s(x,t)$ and $\mu_b(t) = \gamma(t)$.

Figure 2 plots model estimates of these two components for Swedish females in 1875, 1950 and 2000. The senescent component rises linearly from age 25 to about age 75, because in this age range the denominator of the senescent component of (3) is close to 1.0, and the remaining exponential term in the numerator becomes a straight line when plotted on a logarithmic scale as is the case in Figure 2. At ages above about 75 the rate of increase in the force of mortality with age declines in the logistic model and at very high ages $\mu_s(x,t)$ approaches 1.0. The age invariant background component (plotted as horizontal lines in Figure 2) has declined sharply over time from 0.00740 in 1875 to 0.00078 in 1950 and to 0.00013 in 2000. The senescent and background components in Figure 2 add up in each year to the overall model estimate of the force of mortality

plotted in Figure 1. At high ages background mortality is small compared to senescent mortality and it may be ignored for many analytic purposes, especially in contemporary countries with high life expectancy.

b) Trend over time

Trends in adult mortality can be summarized with time series of the three parameters $\alpha(t)$, $\beta(t)$ and $\gamma(t)$. Figures 3a-c present estimated trends in these parameters for each of the 14 countries from 1950 to 2000. Several conclusions can be drawn from these results. There is considerable variation among countries in $\alpha(t)$, but the trend in this level parameter is typically downward. The same is true for background mortality $\gamma(t)$ (see Figure 3c) but there is less variation among countries in $\gamma(t)$ than in $\alpha(t)$. In addition, declines in $\gamma(t)$ are mostly confined to the period 1950 to 1975. After 1975 there appears to be little systematic trend in $\gamma(t)$ in many of these countries, suggesting that background mortality has reached a low level plateau.

The most interesting finding in Figure 3b is that values of $\beta(t)$ are nearly constant for each population. Similar patterns are observed for males (data not shown). This finding confirms earlier observations by Gavrilov and Gavrilova (1991) and Thatcher (1999). In fact, the near constancy of $\beta(t)$ extends further into the past. For example, for Swedish females $\beta(t)$ averaged 0.118 for 1875-1900, 0.119 for 1900-1950 and 0.117 for 1950-2000.

This conclusion about the lack of variation with time in $\beta(t)$ is confirmed in Table 1 which presents averages of annual estimates of the coefficients of variation in $\alpha(t)$, $\beta(t)$ and $\gamma(t)$ for females and males in 14 countries for the period 1950-2000. The coefficient of variation of $\beta(t)$ is small, averaging just 2.2 percent for females and 3.0 percent for males. In contrast, the coefficients of variation for $\alpha(t)$, and $\gamma(t)$ are at least an order of magnitude larger for both males and females. Clearly, $\alpha(t)$, and $\gamma(t)$ are much more variable than $\beta(t)$.

c. Shifting logistic model

The finding that $\beta(t)$ is nearly constant suggests a variant of the logistic model in which this parameter is assumed fixed over time for a population. The senescent component of the standard model (3) then simplifies to

$$\mu_s(x,t) = \frac{\alpha(t)e^{\beta x}}{1+\alpha(t)e^{\beta x}} \qquad (5)$$

in which $\alpha(t)$ is the only time varying parameter. The value of β can differ among populations and may take different values for males and females, but it is constant with respect to time.

A change in the senescent force of mortality from $\mu_s(x,t_0)$ at time t_0 to $\mu_s(x,t)$ at time t is conventionally interpreted as a rise or decline in mortality rates. Formula (5) offers an alternative and unconventional description of changes in the force of mortality. Instead of interpreting mortality as rising or falling over time, the schedule of the force of senescent

mortality can be viewed as shifting to higher or lower ages. This interpretation is possible because (5) has an interesting and very useful property: the age pattern $\mu_s(x,t)$ at time *t* is the same as at an earlier time t_0 except that the function has shifted to higher (lower) ages as senescent mortality falls (rises). The force of mortality at age *x* in year *t* is identical to the value of the force of mortality in an earlier year t_0 at age *x-S(t)* except around age zero. As a result, (5) can be written as

$$\mu_s(x,t) = \frac{\alpha(t_0)e^{\beta(x-S(t))}}{1+\alpha(t_0)e^{\beta(x-S(t))}} \qquad (6)$$

where *S(t)* equals the amount of the shift in years up or down the age axis between t_0 and *t* (equation (6) holds for $x>S(t)$, and $\mu_s(x,t)=0$ for $x<S(t)$). As shown in Appendix A, the conventional up-down and the alternative shifting interpretations are formally equivalent for the logistic model with

$$S(t) = -\frac{\ln(\alpha(t)/\alpha(t_0))}{\beta} \qquad (7)$$

That is, a change in the senescent force of mortality between t_0 and t can be described with (5) as a change in the level parameter $\alpha(t_0)$ to $\alpha(t)$ or equivalently with (6) as a shift by *S(t)* years.

The idea of a shifting mortality schedule can be clarified further by introducing the *senescent life expectancy at birth* denoted $e_s(t)$ and defined as

$$e_s(t) = \int_0^\infty \exp\left\{-\int_0^a \mu_s(x,t)dx\right\}da \qquad (8)$$

It equals the average age at death of a newborn subjected to $\mu_s(x,t)$, assuming no background mortality and no non-senescent mortality at younger ages. The shift to higher or lower ages in the force of senescent mortality function between t_0 and *t* is closely approximated by the change in senescent life expectancy between t_0 and *t*:

$$S(t) \approx e_s(t) - e_s(t_0) \qquad (9)$$

because $\mu_s(x,t)$ is very small around age zero.

The pattern of $\mu_s(x,t)$ given by (6) will be referred to as the *shifting logistic model* . It is a member of a more general class of models for which the *shifting assumption* holds with

$$\mu_s(x,t) = \mu_s(x-S(t),t_0) \qquad (10)$$

The shifting logistic model (6) always implies (10) but the shifting assumption given by (10) may hold even when $\mu_s(x,t)$ does not follow a logistic (more on this below). It should be emphasized that, in general, the shifting property applies only to senescent mortality and not to all adult mortality.

The shifting is evident in Figure 2 where the lines for senescent mortality in 1875, 1950 and 2000 have similar shapes with the schedules for later years moved to higher ages compared to earlier years. The shift equaled 3 years between 1875 and 1950 and 7 years between 1950 and 2000. A shifting pattern for mortality change was proposed earlier by Kanisto et al. (1996) and some of its implications are examined by Bongaarts and Feeney (2002, 2003).

The shifting logistic model describes changes over time in the age pattern of senescent mortality with only one time varying parameter, *S(t)*. This advantage is offset by some loss in the goodness-of-fit. However, the proportion of variance explained by the shifting model with constant β is still an impressive 0.9993 for females and 0.9995 for males (average of 14 populations and all years from 1950 to 1999). The last column of Table 1 presents the R^2 values for males and females in each of the 14 countries with β held constant at its average for 1950-2000. These results are only slightly smaller than the R^2 for the logistic model with a variable $\beta(t)$ presented in the next to last column in Table 1.

These results indicate that the shifting logistic model provides a good general description of age patterns of adult mortality in many countries for the past half century. The next sections of this study examine the implications of this new model.

II. Models for the rate of change in the force of mortality

Past studies of mortality trends have gained important insights by examining the rate of change in the force of mortality with respect to age or time (Horiuchi and Coale, 1990, Horiuchi and Wilmoth, 1998; Keyfitz, 1977; Vaupel, 1986; Vaupel and Romo, 2003). A discussion of this topic is essential to understanding the factors that drive change in adult mortality, to the evaluation of existing projection methods and to the development of a new projection procedure.

a) *Rate of change by age*

The relative derivative of $\mu(x,t)$ with respect to age is defined as

$$k(x,t) = \frac{1}{\mu(x,t)} \frac{\partial \mu(x,t)}{\partial x} \qquad (11)$$

and is referred to as the age-specific rate of mortality change with age (Horiuchi and Coale, 1990) or the life table aging rate (Horiuchi and Wilmoth, 1998).

Figure 4a plots observed and model estimated values of $k(x,t)$ for Swedish females in 1875, 1950 and 2000. The pattern is bell-shaped and varies over time. It is also somewhat different for males than for females (data not shown).

To interpret these changes with age and over time it is useful to decompose $k(x,t)$ into two additive factors representing, respectively, the senescent component $k_s(x,t)$ and the background component $k_b(x,t)$ (see Horiuchi and Wilmoth, 1998, for a slightly different decomposition):

$$k(x,t) = k_s(x,t) + k_b(x,t) \qquad (12)$$

The senescent component $k_s(x,t)$ is defined as the aging rate that would be observed in the absence of background mortality and $k_b(x,t)$ equals the difference between $k(x,t)$ and $k_s(x,t)$.

As shown in Appendix B, for the shifting logistic model

$$k_s(x,t) = \frac{\beta}{1+\alpha(t)e^{\beta x}} = \beta[1-\mu_s(x,t)] \qquad (13)$$

$$k_b(x,t) = \frac{-\beta}{1+[1+1/\gamma(t)]\alpha(t)e^{\beta x}} \qquad (14)$$

Figure 4b plots the model senescent component $k_s(x,t)$ for Swedish females in 1875, 1950 and 2000. At the youngest ages $k_s(x,t)$ is approximately constant and equal to β because $\mu_s(x,t) << 1$. With advancing age $k_s(x,t)$ declines and reaches 0 at very high ages. The schedules for $k_s(x,t)$ and $\mu_s(x,t)$ shift together to higher (lower) ages as senescent life expectancy rises (falls).

Figure 4c plots model estimates of the background component $k_b(x,t)$ for Swedish females in 1875, 1950 and 2000. The value of $k_b(x,t)$ is negative and rises from - β at very young ages to 0 at the oldest ages. An interesting property of the $k_b(x,t)$ schedule is that it shifts to higher/lower ages. But, in general, this shifting occurs at a different rate from the shifting in $\mu_s(x,t)$ and $k_s(x,t)$. In most countries $k_b(x,t)$ either moves slower to the right than $k_s(x,t)$ (when $\gamma(t)$ declines but less rapidly then $\alpha(t)$) or it shifts to the left (when $\gamma(t)$ declines more rapidly then $\alpha(t)$). For Swedish females the background component has clearly moved to the left between 1875 and 1950 and again between 1950 and 2000 due to a very rapid decline in background mortality.

The background and senescent components combine to produce the overall pattern of $k(x,t)$ as shown in Figure 4d for Swedish females in 1950. In general, shifts over time of the ascending portion of the bell shape at lower ages are attributable to shifts in the background component $k_b(x,t)$, and shifts in the descending portion of the bell at higher ages are caused by shifts in the senescent component $k_s(x,t)$ (see related discussion in Horiuchi and Wilmoth, 1998). As a result, the overall bell shaped pattern for $k(x,t)$ exhibits complex changes and can move to the left or right and become wider or narrower depending on trends in $k_s(x,t)$ and $k_b(x,t)$ which in turn are determined by $\alpha(t)$, β and $\gamma(t)$. This makes it difficult to draw conclusions about trends in senescent mortality

from the overall shape of $k(x,t)$. It is therefore preferable to analyze the background and senescent components separately or to limit the analysis to highest ages where senescent mortality dominates.

b) *Rate of change over time*
The relative derivative of the force of mortality with respect to time is defined as

$$\rho(x,t) = -\frac{1}{\mu(x,t)}\frac{\partial \mu(x,t)}{\partial t} \qquad (15)$$

and is called the rate of improvement in mortality (Keyfitz, 1977; Vaupel, 1986; Vaupel and Romo, 2003).

Annual estimates for $\rho(x,t)$ tend to fluctuate widely and the empirical analysis of this variable is therefore usually restricted to averages over periods of one or more decades. Figure 5a plots observed and model estimated values of $\rho(x,t)$ for Swedish females from 1875 to 1950 and from 1950 to 2000. For both periods $\rho(x,t)$ declined with age. To interpret these changes with age and over time it is again useful to decompose $\rho(x,t)$ into two additive factors, the senescent component $\rho_s(x,t)$ and the background component $\rho_b(x,t)$:

$$\rho(x,t) = \rho_s(x,t) + \rho_b(x,t) \qquad (16)$$

The senescent component $\rho_s(x,t)$ is defined as the rate of mortality improvement that would be observed in the absence of background mortality and $\rho_b(x,t)$ equals the difference between $\rho(x,t)$ and $\rho_s(x,t)$.
As shown in Appendix C, if the shifting assumption (10) holds, then

$$\rho_s(x,t) = \dot{e}_s(t)\, k_s(x,t) \qquad (17)$$

$$\rho_b(x,t) = \dot{e}_s(t) k_b(x,t) - \frac{1}{\mu(x,t)}\frac{d\mu_b(t)}{dt} \qquad (18)$$

Where $\dot{e}_s(t)$ denotes the derivative of senescent life expectancy with respect to time: $\dot{e}_s(t) = de_s(t)/dt$. Note that (17) and (18) are valid even if senescent mortality does not follow the logistic, provided the shifting assumption holds. If the shifting logistic model applies, substitution of (13) in (17) gives

$$\rho_s(x,t) = \dot{e}_s(t)\beta[1-\mu_s(x,t)] \qquad (19)$$

and *γ(t)* can be substituted for $\mu_b(t)$ in (18). Equation (19) is a more general version of the formula $\rho_s(t) = \dot{e}_s(t)\,\beta$ derived by Vaupel (1986) for the Gompertz model. (Note also that when background mortality is constant, $\rho(x,t) = \dot{e}_s(t)\,k(x,t)$).

Figure 5b plots model estimates of $\rho_s(x,t)$ obtained from (19) for Swedish females from 1875 to 1950 and from 1950 to 2000. The age pattern of $\rho_s(x,t)$ (but not its level) is the same as for $k_s(x,t)$: at the lowest ages $\rho_s(x,t)$ is constant with age, equal to $\beta \dot{e}_s(t)$, because $\mu_s(x,t) << 1$. With advancing age $\rho_s(x,t)$ declines and reaches 0 at very high ages following the same pattern of relative decline as $k_s(x,t)$. The level of $\rho_s(x,t)$ is substantially higher for 1950-2000 than for 1875-1950 because senescent life expectancy rose at a more rapid pace in the former than in the latter period. The schedule for $\rho_s(x,t)$ shifts to the right as senescent life expectancy rises, as was the case for $k_s(x,t)$ and $\mu_s(x,t)$. Variations in the schedule $\rho_s(x,t)$ over time and with age are therefore the net result of two factors: (1) up/down movements over time due to variation in $\dot{e}_s(t)$ and (2) shifts to higher (lower) ages as $e_s(t)$ rises (falls).

Model estimates of the background component of the rate of mortality improvement for Swedish females from 1875 to 1950 and from 1950 to 2000 are plotted in Figure 5c. Over these two periods the decline in background mortality has been rapid and the second term on the right-hand side of (18) has dominated. This term is directly proportional to the rate of change in background mortality and since *γ(t)* has declined over time (i.e., its derivative is negative) $\rho_b(x,t)$ has been positive, as is evident in Figure 5c. The more rapidly *γ(t)* declines, the more positive $\rho_b(x,t)$ becomes. In addition, $\rho_b(x,t)$ declines sharply with age and approaches zero at high ages. (Note that $\rho_b(x,t)$ is negative when background mortality is constant and senescent life expectancy is rising, because $k_b(x,t)$ is negative.)

The senescent and background components combine to produce the patterns of change in the overall rate of improvement in mortality $\rho(x,t)$ as illustrated in Figure 5d for Swedish females for the period 1950-2000. Below about age 70 the decline in $\rho(x,t)$ with age is attributable to a decline in the background component while the senescent component is approximately constant at $\beta \dot{e}_s(t)$. At ages above 70, $\rho_b(x,t)$ is near zero and $\rho_s(x,t)$ declines, reaching zero at very old ages.

The pattern of $\rho(x,t)$ varies widely over time and among countries as shown in Figure 6a-d which compares model estimates of $\rho(x,t)$ for 1950-1960 and 1985-1995 for England and Wales, France, Italy and Japan. To facilitate the interpretation of these results the values of $\beta \dot{e}_s(t)$ for 1950-1960 and 1985-1995 are plotted as horizontal dashed lines. In the middle adult ages (around age 70) $\rho(x,t)$ is close to this line. At younger ages $\rho(x,t)$ is either above (1950-1960) or below (1985-1995) this line depending largely on the rate of decline in *γ(t)*. At older ages $\rho(x,t)$ declines with age and shifts to higher ages as senescent life expectancy rises.

Given the complexity of changes in $\rho(x,t)$ it is difficult to draw conclusions from them about overall trends in adult mortality. As was the case for $k(x,t)$ it is preferable to analyze the background and senescent components of $\rho(x,t)$ separately. Limiting the analysis to highest ages where the senescent component dominates is somewhat helpful, but it is difficult to determine whether changes at the highest ages are due to shifting (caused by a change in the level of senescent life expectancy) or by an up or down movement (caused by variation in the rate of change in senescent life expectancy).

This analysis of the rate of change in the force of mortality leads to two conclusions. First, the age pattern of $\rho(x,t)$ has changed substantially in recent decades in many countries. This makes it likely that the rate of mortality improvement will not be constant in the future as assumed in the Lee-Carter projection method. Second, the factors responsible for the variation in $\rho(x,t)$ include different trends in background and senescent mortality and the shifting of the pattern of senescent mortality. These findings have implications for projection methods as discussed next.

III. Projecting mortality rates

The models for the force of mortality discussed in the preceding sections will now be applied to gain insights into projection methods. After a brief description of the Lee-Carter method, a new forecasting approach will be proposed.

a) The Lee-Carter method

Lee and Carter (1992) proposed a new statistical method for modeling and forecasting mortality by age which has been adopted widely. For example, the US Census Bureau uses the Lee-Carter forecast as a benchmark for their long-run forecast of life expectancy (Hollman et. al., 2000) and a Social Security Technical Advisory Panel recommended the adoption of the method (Technical Panel on Assumptions and Methods, 1999; Lee and Miller, 2001). Projections of mortality for the G7 countries by Tuljapurkar et. al., (2000) also use this method. Based on the recommendations of an expert group, the United Nations Population Division has prepared its long-range projections to 2300 for all countries in the world with a variant of the Lee-Carter model (United Nations, 2003). Recent discussion of the model and its applications can be found in Lee (1998), Lee (2000), Lee and Miller (2001), Booth et. al. (2002), Carter and Prskawetz (2003) and Tabeau et. al. (2001).

The Lee-Carter method is based on the following mortality model:

$$\ln[m(x,t)] = a(x) + b(x)k(t) + \varepsilon(x,t) \qquad (20)$$

where

$m(x,t)$ = central death rate at age x and time t
$k(t)$ = index of level of mortality
$a(x)$ = age-specific constants describing the general pattern of mortality by age
$b(x)$ = age-specific constants for the relative speed of mortality change
$\varepsilon(x,t)$ = residual

This model provides a good fit to past age-specific mortality rates in the US, explaining 93 percent of the within age group variance between 1900 and 1987 (Lee and Carter, 1992).

Equation (20) provides the basis for making mortality projections. A projection requires only the extrapolation of the index *k(t),* because *a(x)* and *b(x)* are estimated from past data and are held constant for the duration of the projection. An ARIMA time series model is usually used for *k(t)*, and Lee and Carter (1992) and other analysts have assumed a random walk with a drift which describes past trends in *k(t)* well. The implication of assuming a linear trend in *k(t)* to continue into the future is that mortality rates at all ages follow an exponential decline. That is, the projected proportional rate of mortality decline $\rho(x,t)$ in a future year *t* varies by age but it is assumed to be the same at each age as the rate observed in the past:

$$\rho(x,t) = \rho_h(x) \qquad (21)$$

where $\rho_h(x)$ is the observed rate of decline in the death rate at age *x* over some historical period *h* which ends in the base year of the projection. To insure robust results Lee and Carter recommend that estimates of $\rho_h(x)$ be based on historical data for periods of several decades.

The model has several attractive features: a relatively simple demographic model captures the main trends in patterns of past mortality change; forecasting is based on persistent long-term trends and involves no subjective judgment; and the application of statistical time series methods provides probabilistic confidence intervals for the forecast (Lee and Miller, 2001; Booth et. al., 2002). In addition, tests in several populations indicate that projections made with this method are quite accurate over short time horizons (Lee and Miller, 2001). There are, however, also limitations which become increasingly significant as the projection duration rises:
(1) A key assumption of the model (that the rate of decline in mortality at each age remains invariant over time) is violated in several countries in recent decades. Instead of being constant, rates of mortality improvement have tended to decline over time at younger ages while rates of improvement have risen at older ages (Booth et. al., 2002; Carter and Prskawetz, 2003; Lee, 2000; Lee and Miller, 2001). The results presented in Figures 6a-d also indicate that the age pattern of $\rho(x,t)$ has varied over time.
(2) Forecasts in industrialized countries typically underestimate improvements in life expectancy (Lee and Miller, 2001). This is presumably due to a tendency to underestimate mortality improvements at older ages, because errors at young ages have little effect on projections of life expectancy in populations with low mortality.
(3) Extrapolation of trends in age-sex-specific mortality rates or transformations of these rates (Lee-Carter use the log of death rates) can result in implausible results if projected over many decades (Keyfitz, 1984; Lee, 2000; McNown, 1992; Tabeau et al. 2001).

Some investigators have attempted to address these limitations by adding complexity and additional parameters to the Lee-Carter model (Booth et. al., 2002; Carter and Prskawetz, 2003). The alternative approach proposed next provides a simpler solution.

b) A new projection procedure

The shifting model suggests a simple way to project future age-specific rates of senescent mortality from a base year t_0 to a future year *t*:

$$\mu_s(x+S(t),t)=\mu_s(x,t_0) \qquad \text{for } x \geq 25 \qquad (22)$$

(if $S(t)>0$, $\mu_s(x+S(t),t)=0$ for $25 \leq x \leq 25+S(t)$). The projected force of senescent mortality at age $x+S(t)$ in any future year *t* equals the force $\mu_s(x,t_0)$ observed at age *x* in t_0, the base year for the projection. To make a projection for any year *t* the only parameter to be provided is the amount of the shift $S(t)$, and this shift equals the projected rise in senescent life expectancy $e_s(t)$-$e_s(t_0)$ between t_0 and *t* as estimated with (9). The projection of $e_s(t)$ can be based on extrapolation of past trends or on expert opinion. Note that (22) only requires the shifting assumption to hold and does not require that senescent mortality follow the logistic model.

The proposed overall procedure for projecting adult mortality consists of the following steps:

1) Decompose past age-specific rates of period adult mortality into background and senescent components, either annually or for five year periods. Several ways to accomplish this decomposition are available. The preferred approach is to fit the standard logistic model (3) to past age-specific mortality rates to obtain direct estimates of $\gamma(t)$, the level of background mortality . Senescent mortality can then be estimated as $\mu_s(x,t)=\mu(x,t)-\gamma(t)$. An alternative approach is simpler but less accurate: set $\gamma(t)$ equal to the observed mortality rate at age 25-29, on the assumption that senescent mortality is negligible in this age group. Still other approaches may be considered (e.g., making $\gamma(t)$ a simple function of observed mortality rate at ages 25-29), but these will not be evaluated here.

2) Project senescent life expectancy. Once past trends in age-specific senescent mortality are available from step 1, corresponding trends in senescent life expectancy can be calculated with (8). Projection of future trends in $e_s(t)$ can be based on a linear or non-linear extrapolation from these past trends. When projecting over long periods, analysts should also consider expert judgment regarding the plausibility of long range trends and the possibility of convergence to levels observed or projected in countries with the lower mortality.

3) Project senescent mortality by age. Once the future trend in $e_s(t)$ has been projected, age-specific mortality rates in any future year *t* are obtained from (22). The observed age pattern of mortality in the base year t_0 is simply shifted to higher ages with the amount of the shift $S(t)$ equal to the projected increase in senescent life expectancy between *t* and the base year t_0 as estimated in step 2.[2]

4) Project background mortality. Future trends in background mortality $\gamma(t)$ can be extrapolated from past trends estimated in step 1. Linear extrapolation is unlikely to be the best option because $\gamma(t)$ cannot be negative. Assuming an exponential decline at a fixed annual rate should give better results. In contemporary populations with very low background mortality it may be reasonable to assume that $\gamma(t)$ will remain constant in the

future. Several other options (nonlinear trend, approach to a minimum threshold) may be considered.
5) Combine the projections of senescent and background mortality to obtain the overall schedule of adult mortality rates for the duration of the projection. (No attempt will be made to improve on existing methods for projecting mortality under age 25. Conventional methods should be adequate, in particular for countries with high life expectancy because infant and child mortality have reached very low levels and will have little effect on future trends in overall life expectancy).

Figure 7 presents an illustrative application of this procedure for Swedish females. The base year is 2000 and future background mortality is held constant at the level observed in 2000. Two projections are presented with senescent life expectancy rising by 10 and 20 years respectively. The shifting pattern of senescent mortality is evident in Figure 7. No attempt will made here to forecast the year in which these increases in life expectancy are likely to be observed, because linear extrapolation of past trends is potentially problematic and expert opinion should preferably be used for long range projections. However, if the trend in senescent life expectancy is extrapolated linearly from 1950-2000 into the future then a ten year rise will be attained in 2072 and a 20 year rise in 2144.

The new method can also be applied in populations for which mortality data are only available for a single year or period. This is the case for many developing countries where mortality data are often limited. The available information for one year or period provides the baseline estimates of levels of background and senescent mortality. The age pattern of senescent mortality in the base year is then shifted in the projection, but analysts will have to make assumptions about future trends in background mortality and senescent life expectancy.

c) Comparison with Lee-Carter methodology
The new projection method partially addressed the weaknesses in the Lee-Carter method:
a) Variations over time in the age-specific rate of decline of mortality. Lee and Carter (1992) assume the rate of decline in mortality $\rho(x,t)$ projected for age x in some future year t to be constant and equal to the average rate observed in some past period. In contrast, the new method allows this age pattern to change over time by projecting separate trends for background and senescent mortality. As discussed earlier, different trends in background and senescent mortality are a key cause of the changing age pattern of $\rho(x,t)$ over time. In addition, senescent mortality is allowed to shift to higher ages (see next point).
b) Underprojection of improvements in life expectancy. As noted, this is probably in part due to a tendency to underestimate improvements in mortality at older ages. This tendency is not surprising because Lee and Carter (1992) do not take into account the shifting of $\mu_s(x,t)$ and $\rho_s(x,t)$ to higher ages that is explicitly built into the new method. This factor is crucial in long range projections at older ages, because without the shifting, improvements in mortality of the oldest age groups will be underestimated and projections of the population size at these ages will have a downward bias.

c) Implausible age patterns. In long-range projections the assumption of fixed rates of mortality improvements in the Lee-Carter method can produce unsatisfactory results. For example, Figure 8 presents the force of mortality for Sweden projected from 2000 to 2100 and 2250 with the Lee-Carter method. By 2250 the shape of the force of mortality has changed radically. Most analysts would probably consider this pattern unacceptable, because background mortality drops to negligible levels, because mortality declines slightly between ages 60 and 70, and because there is little improvement in mortality at oldest ages. Similar distortions are likely to be encountered in long-range projections for countries where the historical record is short and of poor quality. In contrast, the new method avoids major distortions in future age patterns because the observed base year pattern is preserved and only shifted.

Conclusion

Past age patterns in the force of mortality among adults are well described with a simple logistic model in which the slope parameter is assumed constant over time within each population. The model includes separate components for background and senescent adult mortality which are each summarized with one time-varying parameter. Despite its simplicity, this model captures the main features of complex changes over time in age-specific mortality rates among adults.

The constancy of the slope parameter in this model implies that the senescent component of the force of mortality shifts to higher or lower ages as mortality conditions improve or deteriorate for adults. This shifting model introduces an alternative way of thinking about mortality change. The conventional view is that senescent mortality change implies increases or decreases in age-specific mortality rates. The proposed new view considers mortality decline to be the result of delays in the timing of death. This alternative perspective is captured in the shifting logistic model which provides a parsimonious description of past trends in senescent mortality.

The shifting mortality model also provides the basis for a new method for making projections of age-specific mortality. This method has certain advantages over existing procedures and it will therefore likely produce more accurate long-range projections of adult mortality.

Endnotes

1) Data are available for most years from 1950 to 2000 in the 14 countries, but in several cases data for the last years in the late 1990s or the early 1950s are missing. For details see www.mortality.org. The non-linear least squares routine in STATA was used to obtain estimates of the parameters in the logistic model.

2) Equation (22) assumes that $\mu_s(x+S,t)=0$ for $25 \le x \le 25+S(t)$. This is likely to provide a good approximation in populations in which $\mu_s(x,t_0)$ is very small for $25-S(t) \le x \le 25$. An alternative is to estimate $\mu_s(x+S(t),t)$ for $25 \le x \le 25+S(t)$ by extrapolating senescent mortality from ages above 25+*S(t)*.

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Appendix A: Relationship between level parameter $\alpha(t)$ and the shift in the senescent force of mortality.

A decline in the value of the level parameter from $\alpha(t_0)$ at time t_0 to $\alpha(t)$ at time t implies a decline in senescent mortality from $\mu_s(x,t_0)$ to $\mu_s(x,t)$ as estimated from (5).

Let the ratio of to $\alpha(t)$ to $\alpha(t_0)$ be denoted *p(t)* with

$$p(t) = \frac{\alpha(t)}{\alpha(t_0)} \tag{A1}$$

Substitution of (A1) in (5) gives

$$\begin{aligned} \mu_s(x,t) &= \frac{p(t)\alpha(t_0)e^{\beta x}}{1 + p(t)\alpha(t_0)e^{\beta x}} \\ &= \frac{\alpha(t_0)e^{\beta(x+\ln(p(t))/\beta)}}{1 + \alpha(t_0)e^{\beta(x+\ln(p(t))/\beta)}} \end{aligned} \tag{A2}$$

Define

$$S(t) = -\frac{\ln(p(t))}{\beta} = -\frac{\ln(\alpha(t)/\alpha(t_0))}{\beta} \tag{A3}$$

Substitution of (A3) in (A2) gives

$$\begin{aligned} \mu_s(x,t) &= \frac{\alpha(t_0)e^{\beta(x-S(t))}}{1 + \alpha(t_0)e^{\beta(x-S(t))}} \\ &= \mu_s(x - S(t), t_0) \end{aligned} \tag{A4}$$

A decline in α between t_0 and t is equivalent to a shift of *S(t)* years in the schedule of the force of mortality with *S(t)* given by (A3).

Appendix B : Decomposition of the aging rate for the shifting logistic model

The objective is to derive equations (13) and (14). The first step is to find an equation relating $k(x,t)$ to the parameters in the shifting logistic model. Substitution of (3) in (11) yields

$$\begin{aligned} k(x,t) &= \frac{\dfrac{\partial \mu(x,t)}{\partial x}}{\mu(x,t)} \\ &= \frac{\dfrac{\partial}{\partial x}\left[\dfrac{\alpha(t)e^{\beta x}}{1+\alpha(t)e^{\beta xt}}+\gamma(t)\right]}{\left[\dfrac{\alpha(t)e^{\beta x}}{1+\alpha(t)e^{\beta xt}}+\gamma(t)\right]} \\ &= \frac{\dfrac{\beta\alpha(t)e^{\beta x}}{[1+\alpha(t)e^{\beta x}]^2}}{\left[\dfrac{\alpha(t)e^{\beta x}}{1+\alpha(t)e^{\beta xt}}+\gamma(t)\right]} \\ &= \frac{\beta\alpha(t)e^{\beta x}}{[1+\alpha(t)e^{\beta x}][\alpha(t)e^{\beta x}+\gamma(t)(1+\alpha(t)e^{\beta x})]} \end{aligned} \qquad \text{(B1)}$$

The senescent component $k_b(x,t)$ of $k(x,t)$ is defined as the aging rate that would be observed in the absence of background mortality. Substitution of $\mu_b(x,t)=\gamma(t)=0$ in (B1) gives

$$\begin{aligned} k_s(x,t) &= \frac{\beta}{1+\alpha(t)e^{\beta x}} \\ &= \beta[1-\mu_s(x,t)] \end{aligned} \qquad \text{(B2)}$$

thus confirming (13).

The background component $k_s(x,t)$ of $k(x,t)$ is defined as the difference between $k(x,t)$ and $k_s(x,t)$:

$$k_b(x,t) = k(x,t) - k_s(x,t) \qquad \text{(B3)}$$

Substitution of (B1) and (B2) in (B3) yields

$$k_b(x,t) = \frac{\beta\alpha(t)e^{\beta x}}{[1+\alpha(t)e^{\beta x}][\alpha(t)e^{\beta x}+\gamma(t)(1+\alpha(t)e^{\beta x})]} - \frac{\beta}{1+\alpha(t)e^{\beta x}} \qquad \text{(B4)}$$

Simplification of (B4) gives (14)

Appendix C. Decomposition of the rate of mortality improvement

The aim of this appendix is to derive equations for the senescent and background components of the rate of mortality improvement, provided the shifting assumption (10) holds.

-*Senescent component* $\rho_s(x,t)$.

By definition the senescent component equals the rate of mortality observed when background mortality equals zero, so that

$$\rho_s(x,t) = -\frac{1}{\mu_s(x,t)}\frac{\partial \mu_s(x,t)}{\partial t} = -\frac{\partial \ln \mu_s(x,t)}{\partial t} \qquad \text{(C1)}$$

To derive (17) from (C1) it is necessary first to examine the relationship between $k_s(x,t)$ and $\mu_s(x,t)$ in more detail. The relative derivative of $\mu_s(x,t)$ with respect to age is defined as

$$k_s(x,t) = \frac{1}{\mu_s(x,t)}\frac{\partial \mu_s(x,t)}{\partial x} \qquad \text{(C2)}$$

so that

$$\mu_s(x,t) = \mu_s(0,t)\exp\left[\int_0^x k_s(a,t)da\right] \qquad \text{(C3)}$$

If the shifting assumption (10) holds then changes in $k_s(x,t)$ occur through the same shifts to higher/lower ages as in $\mu_s(x,t)$:

$$k_s(x,t) = k_s(x - S(t), t_0) \qquad \text{(C4)}$$

where *S* equals the amount of the shift in years up or down the age axis $\mu_s(x,t)$ or $k_s(x,t)$ between *t* and t_0. When senescent life expectancy is rising *S(t)* is positive, and (C3) holds for *x>S(t)* with $k_s(x,t) = 0$ for *x<S(t)*; when *S* is negative, (C3) holds for *x*>0). The shift *S* is a function of *t* and t_0, but subscripts will be dropped to simplify the notation. In most populations it is possible to select the base year t_0 so that S is positive because senescent life expectancy has risen between *t* and t_0. The derivation below will assume that this is the case.

With *S(t)*>0, $\mu_s(S(t),t) = \mu_s(0,t_0)$ and substitution of this and of (C4) in (C3) gives

$$\mu_s(x,t) = \mu_s(0,t_0)\exp\left\{\int_{S(t)}^x k_s(a - S(t), t_0)da\right\} \qquad \text{(C5)}$$

for *x>S(t)* and $\mu_s(x,t) = 0$ otherwise.

Substitution of (C5) in (C1) now gives

$$\rho_s(x,t) = -\frac{\partial \ln[\mu_s(0,t_0)\, e^{-\int_S^x k_s(a-S(t),t_0)\,da}]}{\partial t}$$

$$= -\frac{\partial}{\partial t}\int_{S(t)}^{x} k_s(a-S(t),t_0)\,da$$

$$= -\frac{\partial}{\partial t}\int_{0}^{a-S(t)} k_s(y,t_0)\,dy$$

$$= \frac{dS(t)}{dt}k_s(x-S(t),t_0) \qquad \text{(C6)}$$

And substitution of (C4) and (9) in (C6) yields

$$\rho_s(x,t) = \frac{dS(t)}{dt}k_s(x,t)$$

$$= \frac{de_s(t)}{dt}k_s(x,t) \qquad \text{(C7)}$$

thus confirming (17).

-*Background component* $\rho_b(x,t)$.
The background component of the rate of mortality improvement is defined as

$$\rho_b(x,t) = \rho(x,t) - \rho_s(x,t) \qquad \text{(C8)}$$

Substitution of

$$\rho(x,t) = -\frac{1}{\mu(x,t)}\frac{\partial \mu_s(x,t)}{\partial t} - \frac{1}{\mu(x,t)}\frac{\partial \mu_b(x,t)}{\partial t} \qquad \text{(C9)}$$

and of (C7) in (C8) gives

$$\rho_b(x,t) = -\frac{1}{\mu(x,t)}\frac{\partial \mu_s(x,t)}{\partial t} - \frac{1}{\mu(x,t)}\frac{\partial \mu_b(x,t)}{\partial t} - \frac{de_s(t)}{dt}k_s(x,t)$$

$$= \frac{\mu_s(x,t)}{\mu(x,t)}\frac{de_s(t)}{dt}k_s(x,t) - \frac{1}{\mu(x,t)}\frac{\partial \mu_b(x,t)}{\partial t} - \frac{de_s(t)}{dt}k_s(x,t)$$

$$= [\frac{\mu_s(x,t)}{\mu(x,t)} - 1]\frac{de_s(t)}{dt}k_s(x,t) - \frac{1}{\mu(x,t)}\frac{\partial \mu_b(x,t)}{\partial t} \qquad \text{(C10)}$$

The first term on the right hand side of (C10) can be simplified by noting that $d\mu_b(t)/dx = 0$ because background mortality does not vary with age. This implies that

$$\frac{\partial \mu_s(x,t)}{\partial x} = \frac{\partial \mu(x,t)}{\partial x} \tag{C11}$$

and therefore

$$\frac{k(x,t)}{k_s(x,t)} = \frac{\mu_s(x,t)[\partial \mu(x,t)/dx]}{\mu(x,t)[\partial \mu_s(x,t)/dx]} = \frac{\mu_s(x,t)}{\mu(x,t)} \tag{C12}$$

Substitution of (C12) in (C10) with $k(x,t) = k_s(x,t) + k_b(x,t)$ gives

$$\rho_b(x,t) = [\frac{k(x,t)}{k_s(x,t)} - 1]\frac{de_s(t)}{dt}k_s(x,t) - \frac{1}{\mu(x,t)}\frac{\partial \mu_b(x,t)}{\partial t}$$

$$= k_b(x,t)\frac{de_s(t)}{dt} - \frac{1}{\mu(x,t)}\frac{\partial \mu_b(x,t)}{\partial t} \tag{C13}$$

thus confirming (18).

A simpler expression for $\rho(x,t)$ can be obtained if background mortality is constant as appears to be approximately the case over the past two decades in the 14 countries plotted in Figure 3c. With $d\mu_b(t)\,(t)/dt = 0$ the second term on the right side of (C13) disappears. The sum of the senescent and background components then becomes

$$\rho(x,t) = \frac{de_s(t)}{dt}k_s(x,t) + \frac{de_s(t)}{dt}k_b(x,t) = \frac{de_s(t)}{dt}k(x,t) \tag{C14}$$

In this special case the age pattern of $\rho(x,t)$ has the same shape as $k(x,t)$ and the entire schedule of $\rho(x,t)$ is proportional to the rate of change in senescent life expectancy. The three schedules $\mu(x,t)$, $k(x,t)$ and $\rho(x,t)$ maintain their shape over time and shift to higher /lower ages at the same pace as senescent life expectancy rises/falls.

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Table 1: Parameters of the logistic model for adult mortality fitted to observed age-specific death rates for ages 25-109, average of annual estimates for all available years from 1950-2000 in 14 countries.

	$\alpha(t)\times10^{-5}$ (level)		$\beta(t)$ (slope)		$\gamma(t)$ (background)		R^2 Variable $\beta(t)$	R^2 Constant β
	Average	Coef. of variation	Average	Coef. of variation	Average	Coef. of variation	Average	Average
FEMALES								
Austria	0.87	0.310	0.117	0.016	0.00052	0.512	0.9991	0.9991
Canada	1.55	0.292	0.106	0.019	0.00035	0.389	0.9996	0.9996
Denmark	1.52	0.203	0.108	0.042	0.00029	0.658	0.9988	0.9987
England	1.42	0.184	0.109	0.016	0.00027	0.729	0.9997	0.9997
Finland	0.75	0.349	0.119	0.019	0.00053	0.633	0.9991	0.9991
France	0.85	0.443	0.115	0.027	0.00068	0.341	0.9992	0.9991
Italy	0.73	0.346	0.118	0.020	0.00052	0.556	0.9996	0.9996
Japan	0.76	0.628	0.118	0.033	0.00093	0.969	0.9996	0.9995
Netherlands	0.76	0.181	0.116	0.016	0.00035	0.304	0.9994	0.9993
Norway	0.65	0.189	0.117	0.016	0.00032	0.465	0.9992	0.9992
Sweden	0.69	0.290	0.117	0.019	0.00038	0.330	0.9992	0.9992
Switzerland	0.62	0.551	0.120	0.031	0.00047	0.301	0.9991	0.9991
USA	2.18	0.253	0.101	0.018	0.00042	0.183	0.9996	0.9996
W.Germany	0.85	0.228	0.116	0.011	0.00046	0.346	0.9994	0.9994
Average females	**1.01**	**0.318**	**0.114**	**0.022**	**0.00046**	**0.480**	**0.9993**	**0.9993**
MALES								
Austria	2.98	0.215	0.106	0.018	0.00097	0.267	0.9995	0.9994
Canada	3.97	0.333	0.100	0.039	0.00066	0.180	0.9996	0.9995
Denmark	2.66	0.278	0.106	0.039	0.00057	0.296	0.9994	0.9993
England	2.82	0.272	0.107	0.020	0.00032	0.482	0.9995	0.9995
Finland	5.77	0.351	0.099	0.035	0.00088	0.473	0.9994	0.9993
France	4.20	0.249	0.101	0.019	0.00098	0.242	0.9995	0.9995
Italy	2.54	0.332	0.107	0.032	0.00076	0.431	0.9996	0.9996
Japan	2.23	0.366	0.108	0.017	0.00104	0.809	0.9998	0.9998
Netherlands	1.99	0.318	0.109	0.036	0.00042	0.421	0.9996	0.9995
Norway	1.96	0.330	0.109	0.039	0.00067	0.334	0.9996	0.9995
Sweden	1.48	0.299	0.112	0.030	0.00073	0.207	0.9997	0.9996
Switzerland	1.80	0.408	0.111	0.035	0.00090	0.236	0.9994	0.9994
USA	6.36	0.412	0.094	0.041	0.00087	0.348	0.9998	0.9996
W.Germany	2.92	0.173	0.105	0.017	0.00070	0.297	0.9998	0.9998
Average males	**3.12**	**0.310**	**0.105**	**0.030**	**0.00075**	**0.359**	**0.9996**	**0.9995**

Source: Estimated from data in Human Mortality Databank

Figure 1: Age-specific death rates, observed and estimated with logistic model, Swedish females

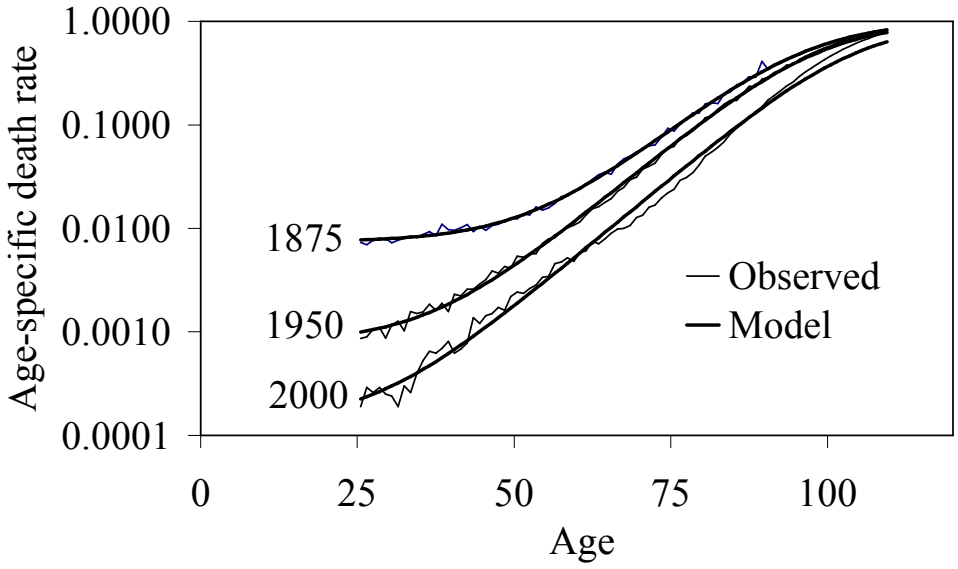

Figure 2: Model estimates of senescent and background death rates rates by age, Swedish females

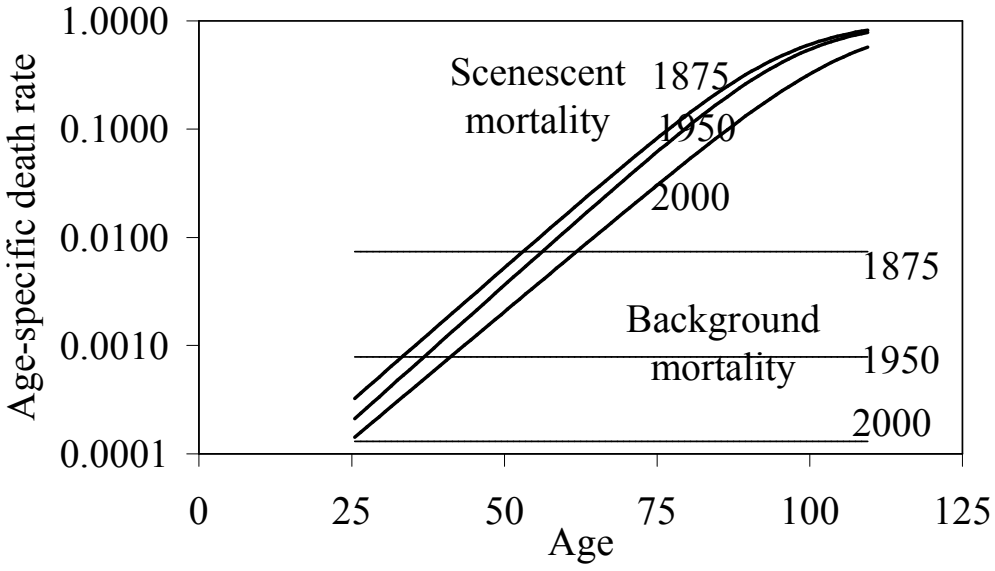

Figure 3a: Estimates of level parameter α in the logistic model for 14 countries, females, 1950-2000

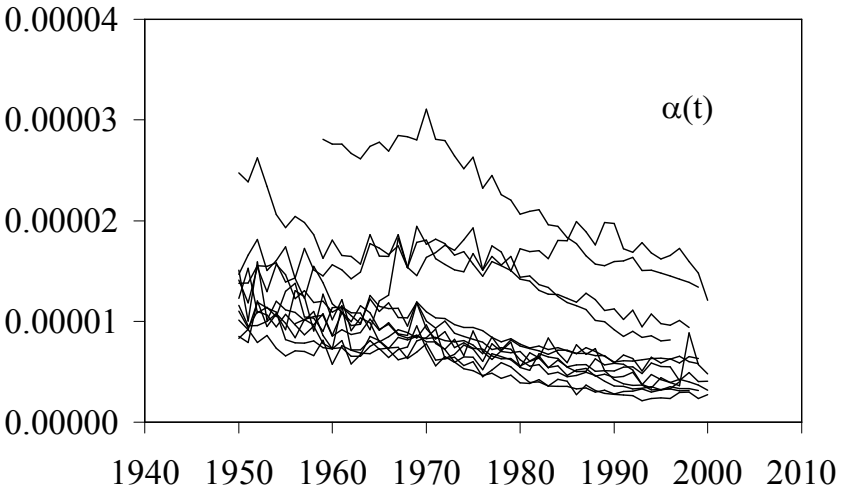

Figure 3b: Estimates of slope parameter β in the logistic model for 14 countries, females, 1950-2000

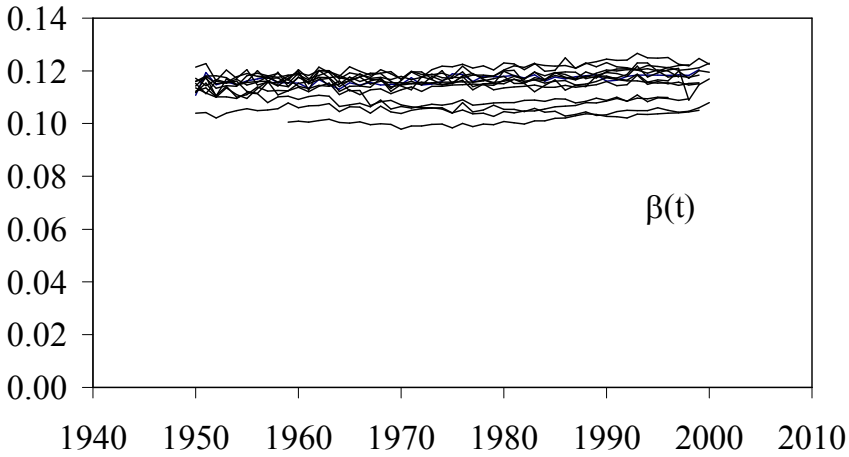

Figure 3c: Estimates of background parameter γ in the logistic model for 14 countries, females, 1950-2000

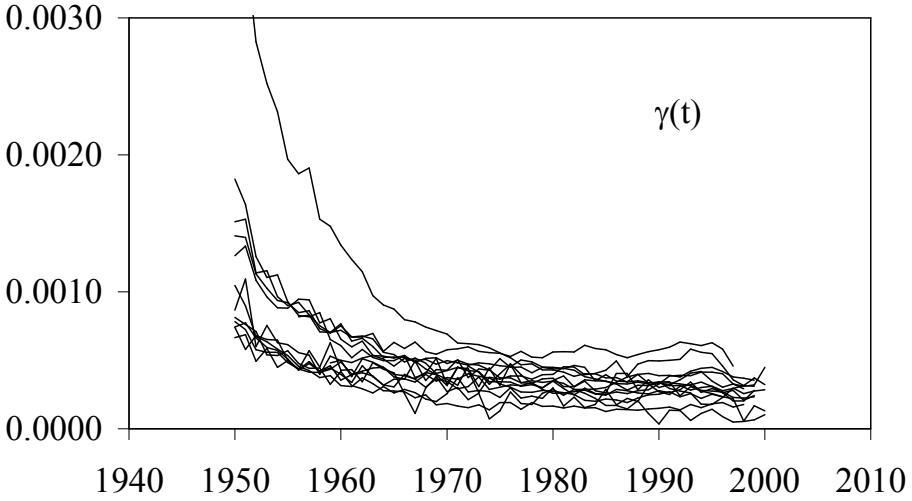

Figure 4a: Life table aging rate, observed and estimated with shifting logistic model, Swedish females

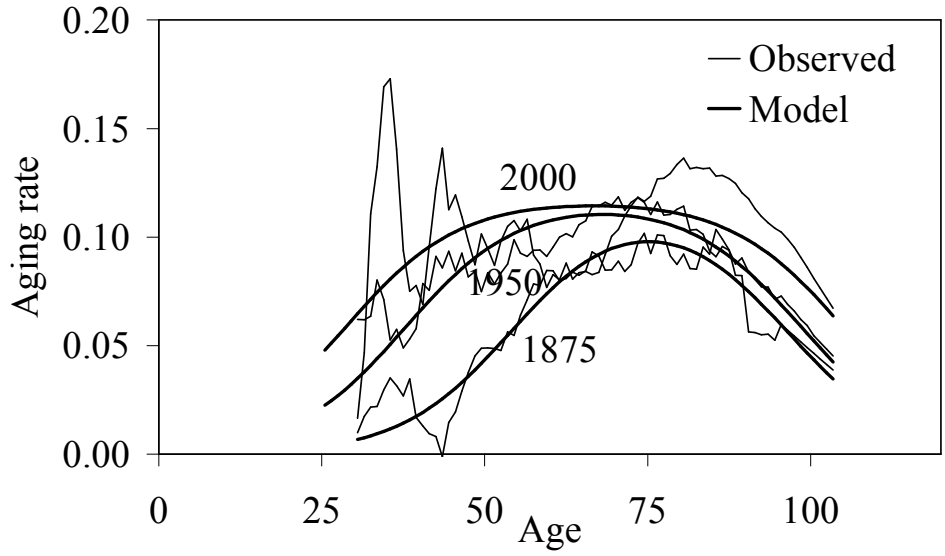

Figure 4b: Senescent component of life table aging rate estimated with shifting logistic model, Swedish females

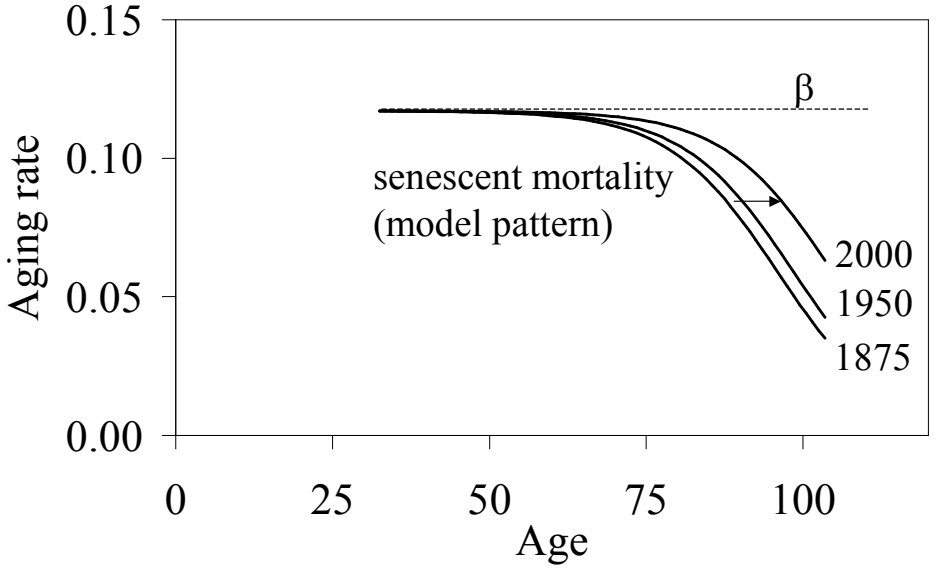

Figure 4c: Background component of life table aging rate estimated with shifting logistic model, Swedish females

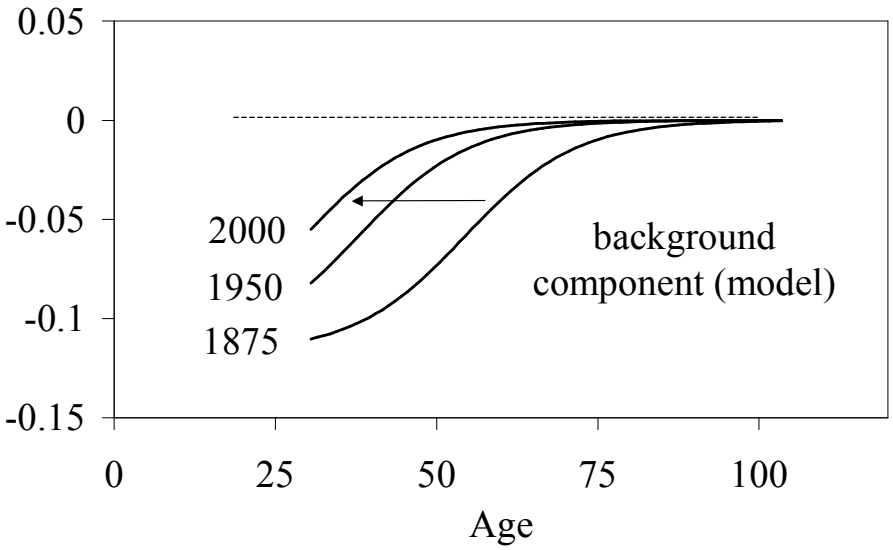

Figure 4d: Senescent and background life table aging rate estimated with shifting logistic model, Swedish females,

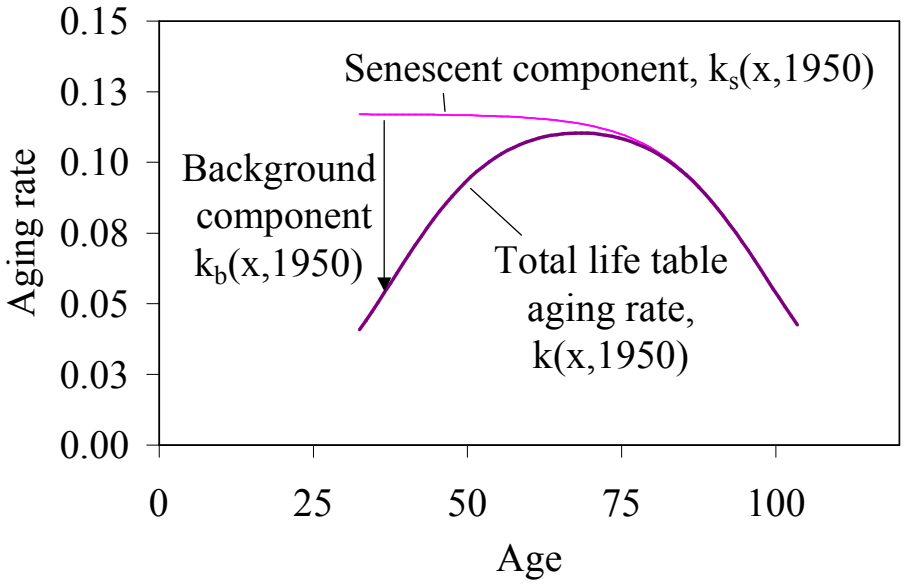

Figure 5a: Rate of mortality improvement, observed and estimated with shifting logistic model, Swedish females

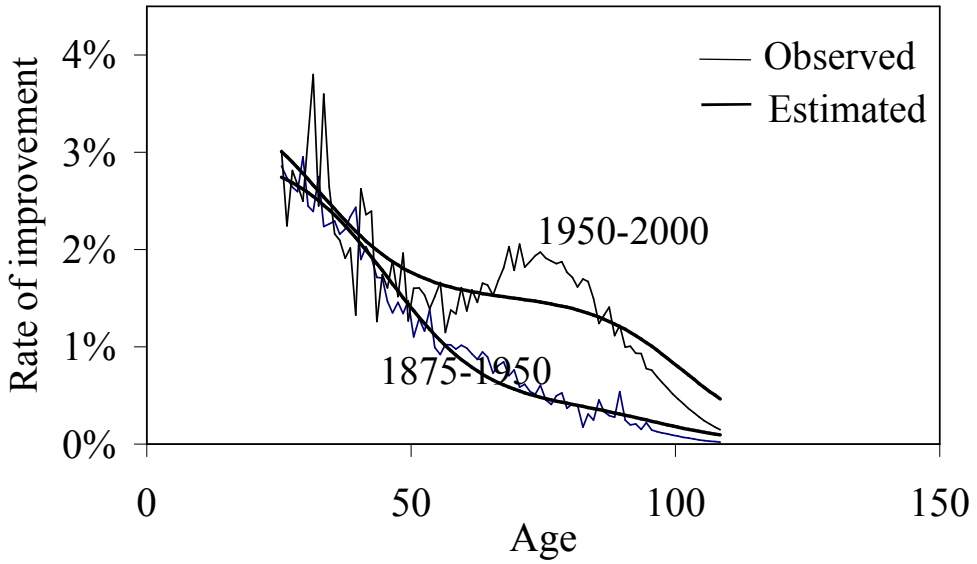

Figure 5b: Senescent component of rate of mortality improvement estimated with shifting logistic model, Swedish females

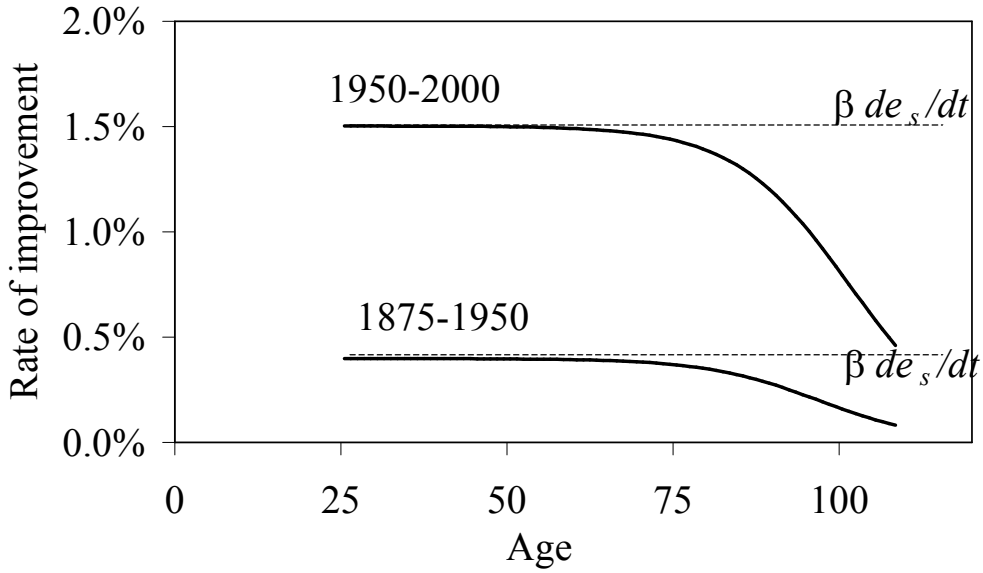

Figure 5c: Background component of rate of mortality improvement estimated with shifting logistic model, Swedish females

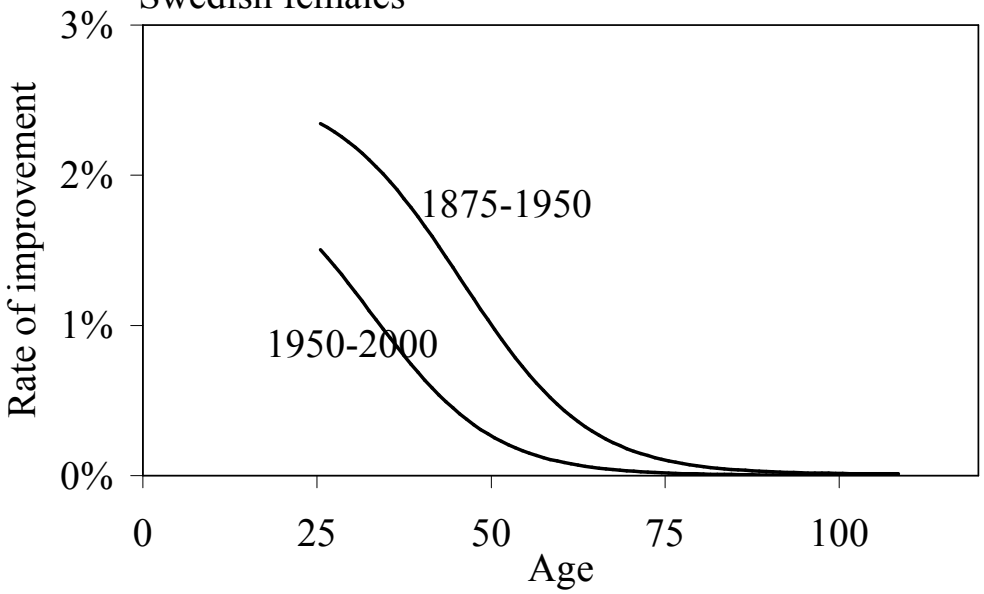

Figure 5d: Decomposition of model estimated rate of mortality improvement, Swedish females

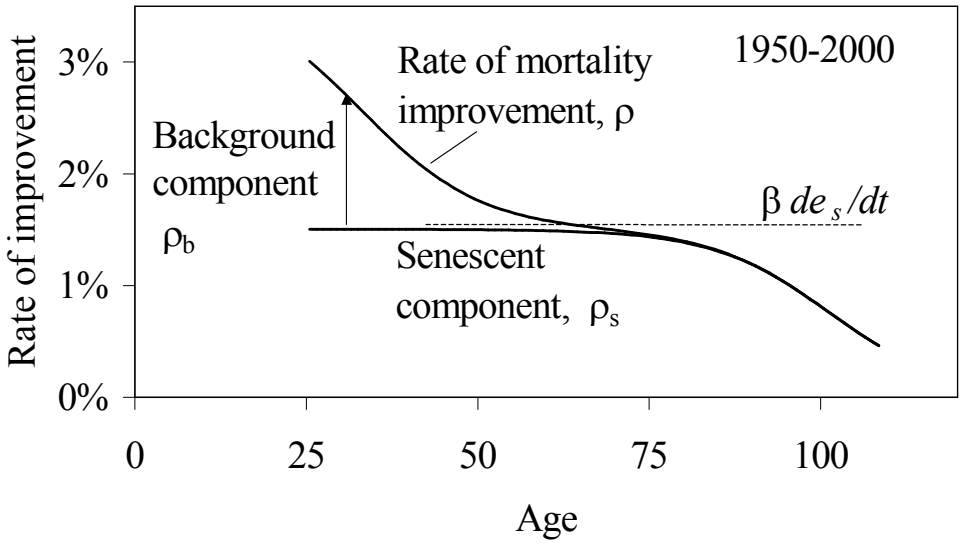

Figure 6a: Rate of mortality improvement estimated with shifting logistic model, females

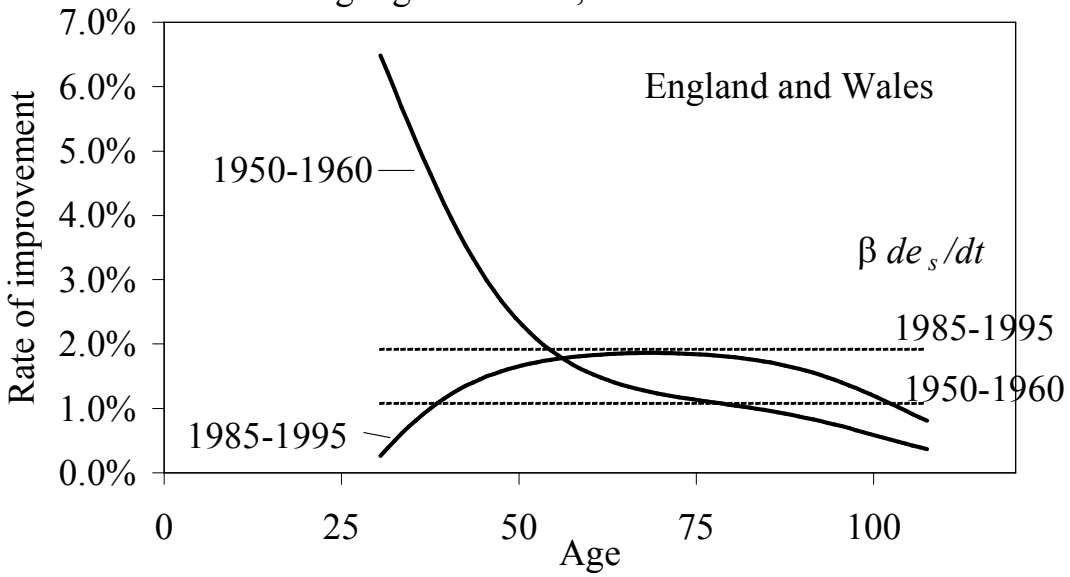

Figure 6a: Rate of mortality improvement estimated with shifting logistic model, females

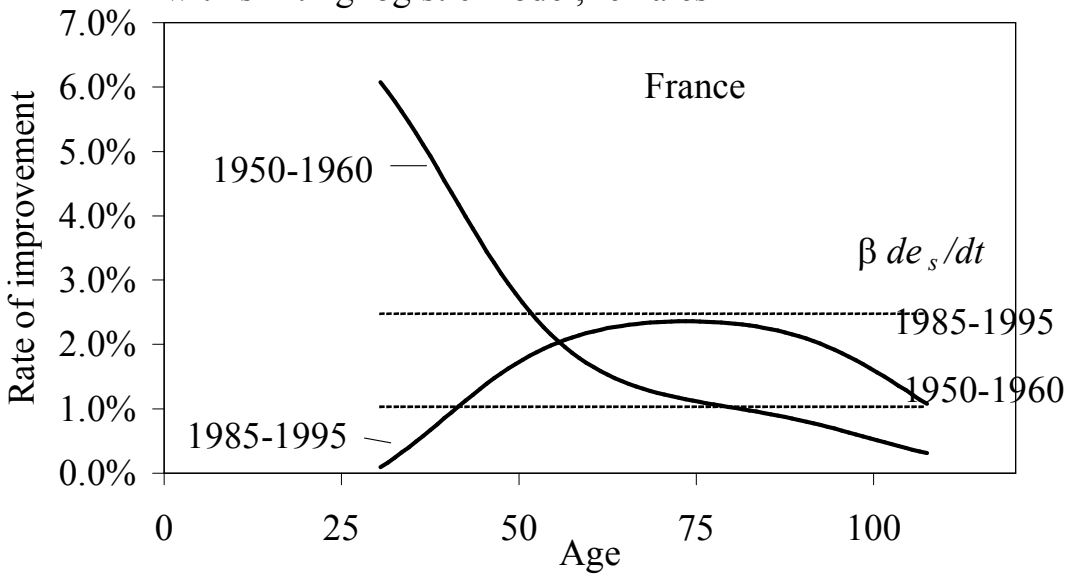

Figure 6c: Rate of mortality improvement estimated with shifting logistic model, females

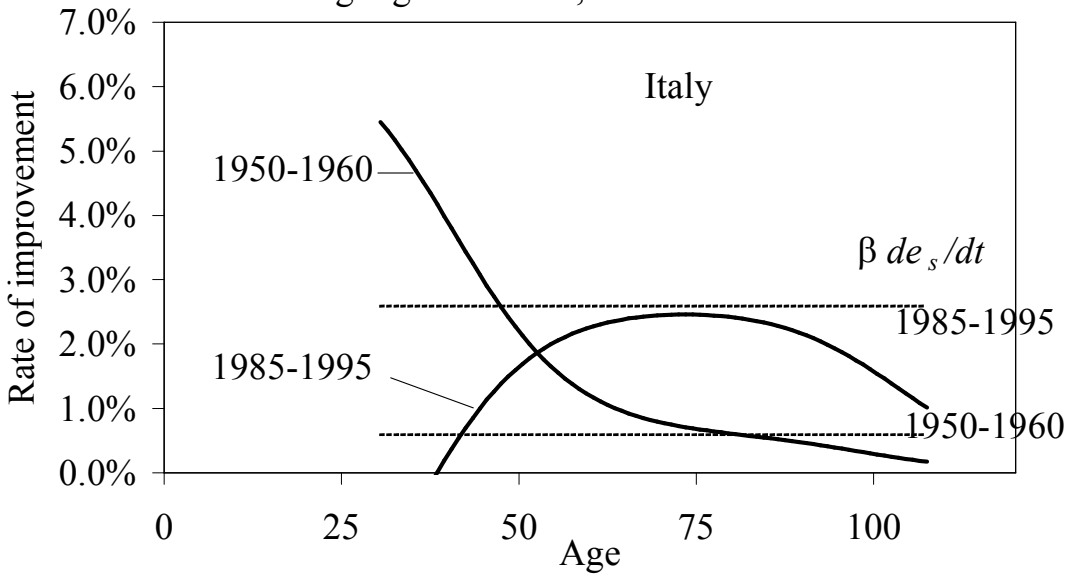

Figure 6d: Rate of mortality improvement estimated with shifting logistic model, females

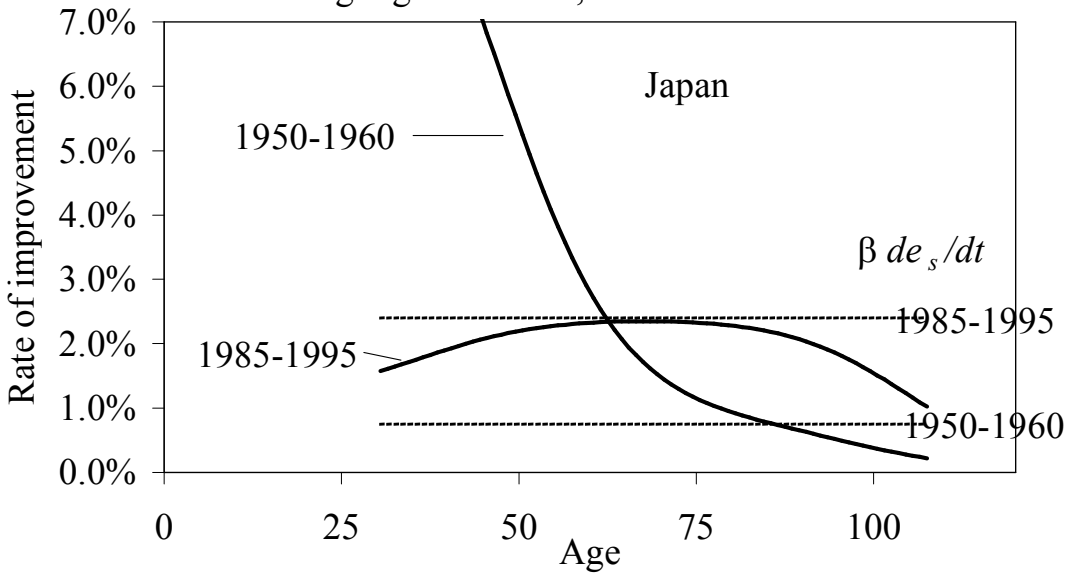

Figure 7: Hypothetical projection by shifting mortality rates observed in 2000 by 10 and 20 years, Swedish females

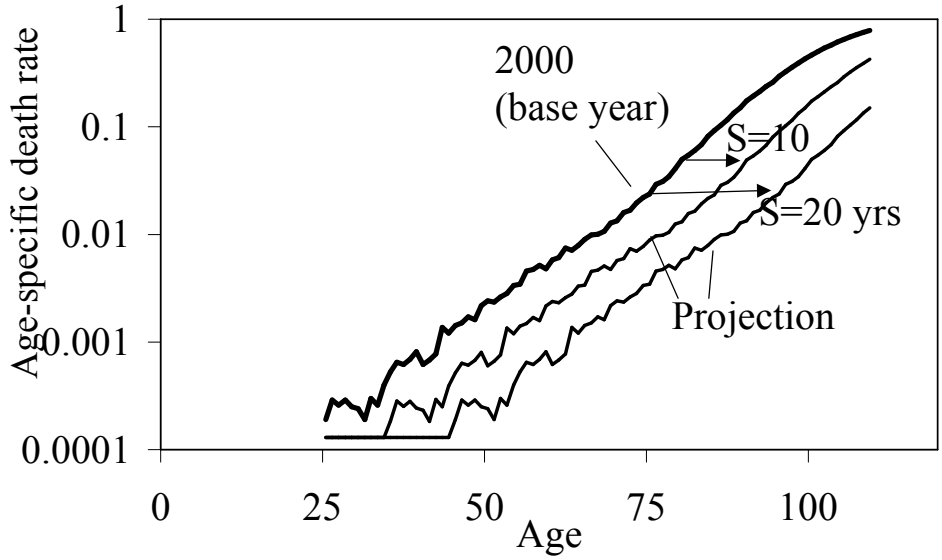

Figure 8: Projections of death rates with Lee-Carter method from 2000 to 2100 and to 2250, Swedish females