SURVIVAL PROBABILITY INDICES OF PERIOD TOTAL FERTILITY RATE:[1]

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ABSTRACT

The tempo effect on period total fertility rate (PTFR) measured by the conventional index includes two elements, *genuine tempo bias* and *spurious tempo bias*. Genuine tempo bias exists in any measure of PTFR when the tempo of fertility changes, while spurious tempo bias exists in the conventional index of PTFR. We identify the cause of spurious tempo bias, and propose two survival probability indices of PTFR that eliminate it. We also introduce an index which improves the "adjusted" TFR introduced by Bongaarts and Feeney. An application of the indices to Japanese fertility data for the past 20 years is presented.

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1. INTRODUCTION

This article focuses on the tempo effect in the measurement of period total fertility rate (PTFR) from a new angle. According to Namboodiri (1991), *total fertility rate* (TFR) is the estimate of "the number of children a women would have during her life time if she were to bear children at successive ages at the same rate as the age-specific fertility rate 'observed' during a particular period." (page 319) TFR thus implies PTFR. This measure has been equated with the sum, or the integral in continuous time, of age-specific fertility incidence rates for a given year over the range of reproductive ages. Below, we refer to PTFR measured in this way as the *conventional* TFR, and denote it by TFR_{CONV}. This article makes a distinction between the concept of PTFR and TFR_{CONV} because the measure does not exactly reflect the concept. TFR_{CONV} suffers from two major problems.

One of the problems, which is known, is that the incidence rate of period fertility for each parity does not reflect a probability distribution of a nonrenewable event. The set of incidence rates reflects a probability distribution of a nonrenewable event if and only if, except for an adjustment needed for censored observations, we can regard people initially at risk of experiencing the event as the common denominator of the rates, and, therefore, the sum of the incidence rates and the probability of never experiencing the event is 1. While the cohort fertility incidence rate for a given parity satisfies this condition, the period fertility incidence rate for a given parity does not because there is no common denominator for the latter. TFR_{CONV} for the first childbirth sometimes exceeds 1, while conceptually it must not, simply because the period incidence rate does not reflect a probability distribution of a nonrenewable event. Nevertheless, TFR_{CONV} is based on a formula which is valid as an index of PTFR only either if the incidence rate reflects a probability distribution of a nonrenewable event, or if the incidence rate at each age is a hazard rate of the Poisson process for a renewable event (Krishnamoorthy 1979), which makes the interpretation of TFR_{CONV} by parity meaningless.

The second problem of TFR_{CONV} is not recognized in the literature and is the major point of this article. In the presence of the *tempo effect* on PTFR, the use of TFR_{CONV} suffers from what we call *spurious tempo bias*. We show in this article that the tempo effect on TFR_{CONV} in fact includes two elements which we call *genuine tempo bias* and *spurious tempo bias.* By *genuine tempo bias*, we mean the difference between complete cohort fertility rate (CFR) and PTFR which exists when there is a systematic change in the tempo of fertility over time. We can assess the amount of genuine tempo bias only under a strong assumption typically made to derive an equation for "demographic translation" (Ryder 1964). By *spurious tempo bias*, we mean bias in TFR_{CONV} as an index of PTFR due to a systematic distortion in the *implicit hazard rate* derived from the period incidence rate in the presence of tempo change. Spurious tempo bias can be eliminated without making any strong assumption by using the true period hazard rate rather than the implicit hazard rate. We prove in this article that spurious tempo bias inflates genuine tempo bias, and the extent of inflation is especially large for the TFR of low parities, and, therefore, the elimination of spurious tempo bias is crucial for measuring accurately the rates of first and second childbirths.

A related but distinct issue, namely the "tempo adjustment" for TFR advocated by Bongaarts and Feeney (1998) (hereafter B-F), has been a subject of controversy. Several scholars (e.g. van Imhoff and Keilman 2000; Kim and Schoen, 2000; Imhoff 2001; Inaba 2003) have pointed out serious problems of the B-F method, while others consider it as useful although it has limitations or needs further elaborations (Yi and Land 2001 2002; Kohler and Philipov 2001). The issue of tempo effect originated in a study by Ryder (1964), who showed a systematic deviation of PTFR from CFR when the tempo of fertility changes over time by a well-known equation for demographic translation. The controversy arose because Bongaarts and Feeney advocated the use of an equation that was similar to Ryder's equation as a method of adjusting PTFR for the sake of measuring PTFR itself, rather than as a method of using PTFR to infer CFR. Although Bongaarts and Feeney define adjusted TFR as a counterfactual index of PTFR in the absence of the tempo effect when it is in fact present, the ambiguity about what the adjusted TFR is measuring has been a focus of debates between Kim and Schoen (2000) and Bongaarts and Feeney (2000) and is also at the heart of Imhoff's (2001) criticism of the B-F method.

As we stated earlier, our major point is that tempo bias in TFR_{CONV} in fact includes two distinct elements --- genuine tempo bias and spurious tempo bias. We will demonstrate that the B-F method is highly problematic partly because it is built upon TFR_{CONV}, which seriously suffers from spurious tempo bias, while the issue with which the B-F study is concerned is really an issue of "adjusting" for genuine tempo bias. We clarify this argument of ours later.

In this article, we examine the properties of two survival probability indices of period fertility rate, which we denote TFR_{SUV_N} and TFR_{SUV_R}, both of which are free from spurious tempo bias, fix the problem that the period incidence rate of fertility does not reflect a probability distribution of a nonrenewable event, and reflect the concept of PTFR as it is defined by Namboodiri. While these indices of TFR each make an additional assumption about a group of women who are at risk of childbearing for a given parity, they do not make any other additional assumptions beyond those explicit in the definition of PTFR. TFR_{SUV_N} and TFR_{SUV_R} differ only in the additional assumption about who is at risk. While TFR_{SUV_R} treats the multiple childbirths of each woman as the outcomes of a repeatable event, TFR_{SUV_N} treats the childbirth of each order as a separate, nonrepeatable event. TFR_{SUV_R} is a slight modification of the parity- and age-specific TFR (PATFR) described by Rallu and Toulemon (1994).

The survival probability indices of PTFR do not eliminate the genuine tempo bias of period fertility rate as indicators of CFR. However, we show under the assumption typically made to derive an equation for demographic translation that inequality CFR ≥ TFR_{SUV_N} ≥ TFR_{CONV} or CFR ≤ TFR_{SUV_N} ≤ TFR_{CONV} holds. Hence, TFR_{SUV_N} is always closer to CFR than TFR_{CONV} is, as we can expect from the fact proven later that spurious tempo bias inflates genuine tempo bias. Here genuine tempo bias is the difference between CFR and TFR_{SUV_N} and spurious tempo bias is the difference between TFR_{SUV_N} and TFR_{CONV}, while the overall tempo bias is the difference between CFR and TFR_{CONV}. The same inequality holds only for the first child for the other index, TFR_{SUV_R}. No simple formal relationship can be derived between CFR and TFR_{SUV_R} for second and later childbirths, however.

We can also derive "adjusted" TFR_{SUV_N} as the alternative to the B-F index. In addition to the fact that adjusted TFR_{SUV_N} is free from spurious tempo bias, we can expect it to be less volatile than the B-F index because TFR_{SUV_N} is always closer than TFR_{CONV} to CFR. As we discuss later, however, adjusted indices -- both the B-F index and the new one -- rests on a restrictive alternative assumption about a synthetic cohort and, therefore, require caveats in their interpretations.

We present our analyses step by step in the following sections. First, we explain the cause of spurious tempo bias in TFR_{CONV}. Second, we present a formal analysis of relationship between CFR, TFR_{CONV}, and TFR_{SUV_N} by assuming a hypothetical situation employed by Ryder (1976), Keilman (1994), and Yi and Land (2002) in deriving an equation for demographic translation, and we analyze the extent to which spurious tempo bias in TFR_{CONV} inflates genuine tempo bias. We also present a limited formal analysis for another index, TFR_{CONV_R}. Third, we discuss the issue of "tempo adjustment" in relationship to the B-F index and the new adjusted index we introduce. Fourth, we describe the empirical measurement of TFR_{SUV_N} and TFR_{SUV_R}. Finally, we apply TFR_{SUV_N}, TFR_{SUV_R}, TFR_{CONV}, adjusted TFR_{SUV_N}, and adjusted TFR_{CONV} (which is the B-F index) to an analysis of recent trends in PTFR in Japan and show, in particular, that the elimination of spurious tempo bias by the survival probability indices greatly reduces tempo bias in TFR_{CONV}.

ON THE SPURIOUS TEMPO BIAS OF THE CONVENTIONAL TFR

The spurious tempo bias of TFR_{CONV} implies that over and above the tempo effect on PTFR which exists for any index of PTFR, TFR_{CONV} suffers from the tempo effect on the implicit hazard rate derived from the period incidence rate. We formally prove this below. Note that while incidence rate has the total number of women at each age as its denominator, hazard rate here implies the rate whose denominator is the number of women who have not yet experienced a childbirth of a given order.

Let $f(t,x)$ be the age-specific fertility incidence rate of a given order at discrete age x and discrete year t. Let $F(t,x)=\sum_{u=1}^{x} f(t,u)$ be the *cumulative* rate, where age 1 indicates the beginning of *reproductive ages*. Let us assume that there has been a systematic shift in the distribution of ages at childbirth toward older ages over time that generates $F(t-1,x) > F(t,x)$. What we call the *implicit hazard rate* is $f(t,x)/(1-F(t,x-1))$ because it is the hazard rate implicitly assumed when TFR_{CONV} is defined as the value of $F(t,x)$ at the end of reproductive age – though implicit hazard rate cannot be defined properly when $F(t,x)$ exceeds 1.[2] The implicit hazard rate, however, differs from the true hazard rate for which the denominator, $S(t,x)$, is the proportion of the corresponding

[2] Note that *f(t,x)* summed across parities may also be regarded as the hazard rate of a Poission process in interpreting TFR_{CONV}. The implicit hazard rate here is based on the interpretation of TFR_{CONV} by parity as a nonrenewable event. We thank Hisashi Inaba for pointing out this fact.

cohort of women who have not yet borne a child of a given order by the end of year t-1 and age x-1. $S(t,x)$ satisfies the following inequality because $F(t,x)$ is assumed to be a monotonically decreasing function of t for a fixed x.

$$\begin{aligned} S(t,x) &= 1-\sum_{u=1}^{x-1} f(t-x+u,u) = 1-\sum_{u=1}^{x-1}[F(t-x+u,u)-F(t-x+u,u-1)] \\ &= 1-F(t-1,x-1)+\sum_{u=1}^{x-2}[F(t-x+u+1,u)-F(t-x+u,u)] < 1-F(t-1,x-1). \end{aligned} \quad (1)$$

It follows that

$$f(t,x)/S(t,x) > f(t,x)/[1-F(t-1,x-1)] > f(t,x)/[1-F(t,x-1)] \quad (2)$$

and, therefore, the true hazard rate is always greater than the implicit hazard rate that is assumed in the calculation of TFR_{CONV} when there is a systematic tempo change that generates a monotonic decrease of $F(t,x)$ as a function of t. Equation (1) indicates that this distortion in the implicit hazard rate depends on the accumulation of past tempo effects, and not just on the tempo change between time t-1 and t. Equation (2) indicates that the hazard rate of event occurrence at each age x is systematically underestimated in the calculation of TFR_{CONV}, and this causes an underestimation of true PTFR by TFR_{CONV}. Similarly, we can prove that TFR_{CONV} systematically overestimates true PTFR when there is a systematic shift of ages at childbirth toward younger ages over time. An important fact is that this direction of deviation in TFR_{CONV} from true PTFR is the same as that generated by the tempo effect on the deviation of TFR_{CONV} from CFR, as shown later. We thus call this problem spurious tempo bias in TFR_{CONV}.

In conclusion, the tempo effect on implicit hazard rate assumed in the calculation of TFR_{CONV} yields spurious tempo bias. Spurious tempo bias can be eliminated by using the period hazard rate based on the estimate for the corresponding cohort population of women at risk for its denominator. In the next section, we formally investigate the magnitude of spurious tempo bias compared with genuine tempo bias.

A FORMAL ANALYSIS

1. A Review of the Theoretical Background

This article does not intend to present an extensive formal analysis for various situations that can be modeled mathematically, but to illustrate the importance of spurious tempo bias. Hence, we consider the simplest situation assumed in deriving the basic formula of demographic translation (Ryder 1964; Keilman 1994; Yi and Land 2002).

While TFR_{CONV} is based on *period incidence rate*, two measures of PTFR that we propose in this article, TFR_{SUV_N} and TFR_{SUV_R}, are both based on the *period hazard rate* whose denominator is restricted to women who are at risk. When their formulas are applied to cohort fertility, these three indices are equivalent. The three indices are not equivalent for period fertility, however. We will show below that the inequality CFR ≥ TFR_{SUV_N} ≥ TFR_{CONV} or CFR ≤ TFR_{SUV_N} ≤ TFR_{CONV} holds, which indicates that TFR_{SUV_N} is always to closer than TFR_{CONV} to CFR, under an assumption made to derive an equation for demographic translation. We then assess the relative extent of spurious tempo bias. We also present a limited formal analysis for TFR_{SUV_R}.

We assume for the formal analysis in this section a situation where (i) women are not subject to death during their reproductive ages, (ii) CFR is constant over time, and (iii) the distribution of ages at childbirth is shifting, by a constant amount of *r**, over each cohort without changing the shape of the distribution. We call *r** the *cohort effect on tempo change*. Unless specified otherwise, the analysis described below applies to each parity separately even though we omit the subscript that indicates the parity.

We employ below a continuous-time expression because it is more convenient for expressing tempo change, and, following the notation of Yi and Land (2002), we denote by $g_p(t, x)$ the period fertility incidence rate at year *t* and reproductive age *x*, and by $f_{c,T}(x)$ the fertility incidence rate at reproductive age *x* for a cohort of women with birth year *T.* By definition $g_p(t,x) = f_{c,t-x}(x)$, $\mathrm{TFR}_{\mathrm{CONV}}(t) \equiv \int_0^{x_{\mathrm{MAX}}} g_p(t,x)dx$ for the *reproductive* age range [0, x_{MAX}], and $\mathrm{CFR}(T) \equiv \int_0^{x_{\mathrm{MAX}}} f_{c,T}(x)dx$. By assumption (ii), CFR(*T*) does not depend on birth year *T*, and from assumption (iii), we obtain $f_{c,t-u}(x) = f_{c,t}(x + r * u)$ for any year interval *u* when *x+r*u* is within the range of reproductive ages. From these equations, we obtain an expression for demographic translation:

$$\begin{aligned}\mathrm{TFR}_{\mathrm{CONV}}(t) &= \int_0^{x_{\mathrm{MAX}}} f_{c,t-x}(x)dx = \int_0^{x_{\mathrm{MAX}}} f_{c,t}(x + r * x)dx \\ &= [\int_0^{x_{\mathrm{MAX}}} f_{c,t}(u)du]/(1+r^*) \text{ where } u = (1+r^*)x \qquad (3) \\ &= \mathrm{CFR}(t)/(1+r^*) = \mathrm{CFR}/(1+r^*).\end{aligned}$$

Hence, $\mathrm{TFR}_{\mathrm{CONV}}(t)$ does not depend on year *t*. The cohort effect on tempo change *r** here differs from the *period effect on tempo change r* (Yi and Land 2002). As an alternative to assumption (iii), we can assume that (iii)* the distribution of ages at childbirth is shifting, by a constant amount of *r,* over each *period* without changing the quantum and shape of the distribution. Assumption (iii)* means that $g_p(t,x) = g_p(0, x - rt)$, and we then obtain

$$\begin{aligned}\mathrm{CFR} = \mathrm{CFR}(0) &= \int_0^{x_{\mathrm{MAX}}} f_{c,0}(x)dx = \int_0^{x_{\mathrm{MAX}}} g_p(x,x)dx \\ &= \int_0^{x_{\mathrm{MAX}}} g_p(0,(1-r)x)dx = [\int_0^{x_{\mathrm{MAX}}} g_p(0,u)du]/(1-r) \text{ where } u = (1-r)x \quad (4) \\ &= \mathrm{TFR}_{\mathrm{CONV}}(0)/(1-r) = \mathrm{TFR}_{\mathrm{CONV}}/(1-r).\end{aligned}$$

As Yi and Land (2002: page 273) have shown, we obtain from equations (1) and (2) a simple relationship between the cohort effect on tempo change, *r**, and the period effect on tempo change *r*: *r* = *r**/(1+*r**).

2. $\mathrm{TFR}_{\mathrm{SUV\_N}}$

In defining $\mathrm{TFR}_{\mathrm{SUV\_N}}$, we treat the childbirth of each order as a separate *nonrepeatable event.* In defining hazard rate, we therefore assume that everybody who has not yet experienced a childbirth of a given order is at risk of the event. It follows that we define *period fertility hazard rate*, $h_p(t,x)$ to be

$$h_p(t,x) = g_p(t,x)/S_{c,t-x}(x) = f_{c,t-x}(x)/\left[1-\int_0^x f_{c,t-x}(u)du\right], \qquad (5)$$

where $S_{c,t-x}(x)$ is the survival probability of not yet having a childbirth of a given order for women of cohort *t-x* at reproductive age *x* and year *t*. It follows from $f_{c,t-u}(x) = f_{c,t}(x+r*u)$ and equation (7) that

$$\begin{aligned} h_p(t,x) &= f_{c,t}(x+r*x)/\left[1-\int_0^x f_{c,t}(u+r*x)du\right] \\ &= f_{c,t}(x+r*x)/\left[1-\int_0^{x+r*x} f_{c,t}(v)dv\right] \text{ where } v = u+r*x \qquad (6) \\ &= h_{c,t}((1+r^*)x). \end{aligned}$$

Using this hazard rate, and using a well-known relationship between hazard rate and survival probability (Kalbfleisch and Prentice 1980; Yamaguchi 1991), a survival probability index of TFR at year *t*, $\text{TFR}_{\text{SUV\_N}}(t)$, is shown to satisfy the following equation.

$$\begin{aligned} \text{TFR}_{\text{SUV\_N}}(t) &\equiv 1-\exp[-\int_0^{x_{MAX}} h_p(t,x)dx] = 1-\exp[-\int_0^{x_{MAX}} h_{c,t}((1+r^*)x)dx] \\ &= 1-\exp[-(\int_0^{x_{\text{MAX}}} h_{c,t}(u)du)/(1+r^*)] \text{ where } u=(1+r^*)x \qquad (7) \\ &= 1-(1-\text{CFR}(t))^{\frac{1}{1+r^*}} = 1-(1-\text{CFR})^{\frac{1}{1+r^*}} = 1-(1-\text{CFR})^{(1-r)} \end{aligned}$$

Hence, TFR_{SUV}(t) does not depend on year *t*. Although the parameterization slightly differs, this is the alternative equation for demographic translation that Keilman (1994) derived under the same assumption about the hazard rate of nonrepeatable events.

What Keilman did not point out is that the comparison of Keilman's and Ryder's equations allows us to compare $\text{TFR}_{\text{SUV\_N}}$ and TFR_{CONV} and to assess the relative extent of spurious tempo bias. Let *z* = CFR, and *F*(*z*) = TFR_{CONV}-$\text{TFR}_{\text{SUV\_N}}$. Then, from equations (4) and (7), we obtain

$$F(z) = (1-r)z - [1-(1-z)^{(1-r)}] \quad . \qquad (8)$$

Function $F(z)$ satisfies the conditions that $F(0) = 0$, $F'(0) = 0$, $F'(z) = (1-r)[1-(1-z)^{-r}]$ < 0 when $r > 0$, $F'(z) > 0$ when $r < 0$, and $F'(z) = 0$ for $r = 0$ for $z > 0$. Since CFR > TFR_{SUV_N} holds when $r > 0$, and CFR < TFR_{SUV_N} holds when $r < 0$, it follows that CFR > TFR_{SUV_N} > TFR_{CONV} holds when $r > 0$, and CFR < TFR_{SUV_N} < TFR_{CONV} holds when $r < 0$, and the three indices are equal when $r = 0$.

Hence, under assumptions (i), (ii), and (iii) (or (iii)*), TFR_{SUV_N} is always a better indicator of CFR than TFR_{CONV} is. Figure 1 graphically depicts how TFR_{CONV}–CFR and TFR_{SUV_N}–CFR change as a function of CFR. It shows that while the difference between TFR_{CONV} and CFR linearly increases with CFR, the difference between TFR_{SUV_N} and CFR reaches a maximum at $\mathrm{CFR} = 1-(1-r)^{1/r}$ which is close to $1-e^{-1} \doteq 0.542$ in the neighborhood of $r = 0$.

(Figure 1 About Here)

It is also important to assess the *relative* improvement that TFR_{SUV_N} provides as the estimator of CFR compared with TFR_{CONV}. The index for the *proportional reduction in tempo bias* by the use of TFR_{SUV_N}, $I(z,r)$, is given as

$$I(z,r) = \frac{\left|\mathrm{TFR_{SUV\_N}} - \mathrm{TFR_{CONV}}\right|}{\left|\mathrm{CFR} - \mathrm{TFR_{CONV}}\right|} = \frac{\left|F(z)\right|}{\left|r\right| z} \qquad (9)$$

It can easily be shown that $I(z,r)$ monotonically increases with z, $\lim_{z\to 0} I(z,r) = 0$, and $I(1,r) = 1$. The values of $I(z,r)$ do not vary with r greatly when r is small and are approximately equal to $\lim_{r\to 0} I(z,r) = 1 + [(1-z)\log(1-z)]/z$ in the neighborhood of $r = 0$. Figure 2 depicts this function. Since parity-specific CFR decreases with parity, the proportional reduction in tempo bias by the use of TFR_{SUV_N} is greater for smaller parities. For example, if parity-specific CFRs for the first, second, third, and fourth birth are respectively, 0.8, 0.6, 0.4, and 0.2, the proportions of reduction in tempo bias are 0.598, 0.389, 0.234, and 0.107, respectively, for parity 1 through 4 when *r* is near zero, and yield 0.413, which is the weighted average of *I*(z,0) with the value of *z* as weights, as the average proportion of reduction in the tempo bias in TFR_{CONV} by the use of $\text{TFR}_{\text{SUV\_N}}$.

(Figure 2 About Here)

3. $\text{TFR}_{\text{SUV\_R}}$

In defining the alternative index of PTFR, $\text{TFR}_{\text{SUV\_R}}$, we treat the childbirth of successive orders as a *repeatable event* and therefore assume that women who bore exactly (*i*-1) children before age *x* are at risk of bearing the *i*th child at age *x.* $\text{TFR}_{\text{SUV\_R}}(1)$ for the first childbirth is by definition the same $\text{TFR}_{\text{SUV\_N}}(1)$.

We first give an alternative definition for CFR of the *i*th child (where *i* > 1), and we next define $\text{TFR}_{\text{SUV\_R}}$. We can express CFR for birth order *i* using the cumulative rate of bearing a child of birth order *i* at age *x* for cohort *T*, $F_{c,i,T}(x)$, as

$$F_{c,i,T}(x) = \int_0^x f_{c,i-1,T}(u)\left[1 - S_{c,i,T}(x|u)\right]du \qquad (10)$$

$$\text{CFR}_T(i) = F_{c,i,T}(x_{MAX}) \ , \qquad (11)$$

where $S_{c,i,T}(x|u)$ in equation (10) is the *conditional* survival probability of *not* experiencing a birth of the *i*th child by age *x* after having had a birth of an (*i*-1)th child at age *u*. Equations (10) and (11) imply that $\text{CFR}_T(i)$ is the sum, over all reproductive *ages* (*u*), of the product of the probability density of bearing the (*i*-1)th child at age *u* and the conditional probability of bearing the *i*th child before the end of reproductive age given *u* as the age of entry into the risk of bearing the *i*th child. The equivalence of CFR defined by equation (11 with that defined as $\int_0^{x_{\max}} f_{c.i,T}(x)dx$ can be easily proven, and the proof is omitted. An important fact here, however, is that this equivalence requires that the cohort hazard rate of having a birth as a repeatable event be expressed as an unconstrained bivariate function of both age *x* and the time of entry into risk *u*, or equivalently, as an unconstrained bivariate function of age *x* and the duration of risk *x-u.* This fact indicates that for the estimation of PTFR by assuming the childbirth as a repeatable event, it is desirable to express period hazard rate as a bivariate function of both age and the duration of risk. Empirically, however, this expression may be limited in use because of data non-availability and, in addition, the hazard rate estimated for each combination of age and duration of risk may not be stable because its denominator can be very small. A model that assumes only age dependence, and no duration dependence, may still describe the process adequately if age dependence, rather than duration dependence, is dominant. Assuming only age dependence is also consistent with the traditional definition of PTFR and known as the parity- and age-specific TFR (PATFR hereafter) (e.g., Rallu and Touleman 1994).

Hence, for practical reason, we consider below the model where period hazard rate is a function of age and does not depend on the duration of risk. In defining TFR_{SUV_R}, we must express all elements of TFR_{SUV_R} in terms of period hazard rate rather than period incidence rate because the latter suffers from spurious tempo bias as described before. For second or later childbirths, the period hazard rate $h_{p,i}(t,x)$ of order *i* at reproductive age *x* and year *t* is defined, using the incidence rates $f_{c,i,t-x}(x)$ of the cohort with birth year *t-x*, by the equation

$$h_{p,i}(t,x) = \frac{f_{c,i,t-x}(x)}{\int_0^x f_{c,i-1,t-x}(u)du - \int_0^x f_{c,i,t-x}(u)du}, \qquad (12)$$

where the denominator indicates the probability that a cohort of women with birth year *t-x* will have exactly (*i*-1) children before age *x*. $TFR_{SUV\_R}(i,t)$ for $i > 1$ is then defined sequentially by the following set of equations:

$$f*_{p,1}(t,x) = h_{p,1}(t,x)\exp(-\int_0^x h_{p,1}(t,u)du) \quad (13$$

$$f*_{p,i}(t,x) = h_{p,i}(t,x)\left[\int_0^x f*_{p,i-1}(t,u)\exp(-\int_u^x h_{p,i}(t,v)dv)du\right] \quad (14)$$

$$\text{TFR}_{\text{SUV\_R}}(i,t) \equiv \int_0^{x_{MAX}} f*_{p.i-1}(t,x)\left[1-\exp\left\{-\int_x^{x_{MAX}} h_{p,i}(t,u)du\right\}\right]dx, \quad (15)$$

where f^* is the period incidence rate derived from the period hazard rate. Under the assumption of a time-invariant cohort effect on tempo shift such that $f_{c,t-u}(x) = f_{c,t}(x + r*u)$, we obtain $h_{p,i}(t,x) = h_{c,i,t}((1+r*)x)$ (the proof of which is similar to that of equation (6) and is omitted). It follows that $TFR_{SUV\_R}(i,t)$ can be reexpressed, using cohort hazard rate and cohort survival probability. However, we cannot obtain any simple formal relationship between CFR and TFR_{SUV_R}.

4. Notes on the Relative Advantages and Disadvantages of the Indices of TFR

Regarding the choice between the two survival-probability indices of TFR, the major advantage of TFR_{SUV_R} over TFR_{SUV_N} is that the former employs the definition of hazard rate which eliminates one major aspect of population heterogeneity, namely, the difference in risk between those with (i-1) children and those with fewer children in experiencing childbirth of the ith order at each age. This definition of hazard rate is also consistent with many empirical studies of childbirth based on hazard rate models.

However, there are four disadvantages of TFR_{SUV_R} compared with TFR_{SUV_N}. First, unlike TFR_{SUV_N}, TFR_{SUV_R} does not have a simple formal relationship with CFR. Second, strictly speaking, spurious tempo bias is *eliminated* only by the use of TFR_{SUV_N} except for the first childbirth, for which TFR_{SUV_R} = TFR_{SUV_N} because the logic about the the tempo bias in the implicit hazard rate of TFR_{CONV} applies in comparison with the hazard rate of TFR_{SUV_N}. Third, the hazard rate of childbirth as a repeatable event generally depends not only on age but also on the duration of risk after the previous birth for second and later births. While the definition of PTFR considers only age dependence, duration dependence over and above age dependence is empirically present, and therefore, assuming only age dependence may impose an undesirable constraint on the underlying stochastic process implicitly modeled in providing TFR_{SUV_R}. On the other hand, the only time dimension for the childbirth of each parity as a nonrepeatable event is age, and, therefore, no similar problem occurs for TFR_{SUV_N}. Note that since age dependence is usually very strong empirically (Yamaguchi and Ferguson 1996) for second and later childbirths, Henry's and Feeney's life table index of TFR (Henry (1980[1953]) and Feeney (1983)) which considers only duration dependence can be criticized for the opposite reason that it ignores age dependence. Fourth, empirical estimates for hazard rate in the calculation of TFR_{SUV_R} may not be stable at young ages for second and later births because a very small population of women will be at risk – although the effect of such instability on TFR_{SUV_R} is usually small because unstable hazard rates are applied only to those who are at risk.

ON THE "ADJUSTMENT" OF PERIOD TOTAL FERTILITY RATE

Bongaarts and Feeney (1998) introduced "adjusted TFR" for the case where the period effect on tempo change, *r*, varies with period *t*, so that we have *r*(*t*) instead of *r*, and defined $TFR_{CONV}(i)/(1-r(i,t))$ for each parity *i* at year *t* to be adjusted TFR(*i*).

Formally, when we assume time-varying period effects on tempo change without changing the quantum and shape of the distribution, we obtain $g_p(t,x) = g_p(0, x - \int_0^x r(t)dt)$ and this does not permit any simple relationship between CFR and TFR_{CONV} because the average rate of shift $\left[\int_0^x r(t)dt\right]/x$ changes with age x.

Bongaarts and Feeney, however, proposed their adjusted TFR not as an indicator of CFR, but as a counterfactual measure of *current* PTFR which "would have been observed in year *t* had there been no change in age at childbearing, i.e., if *r*(*t*) had been zero (page 289)". Note first that B-F's conceptualization of PTFR differs from the traditional definition of PTFR. PTFR is traditionally based on the idea of a synthetic cohort where, since $g_p(t,x) = f_{c,t-x}(x)$, cohort changes with age, while year *t* is fixed. On the other hand, since $f_{c,T}(x) = g_p(T+x,x)$, age changes with year, while cohort is fixed, in the fertility rate of real cohorts. Bongaarts and Feeney consider a situation where women postpone or accelerate childbirth timing where age changes with year, and not with cohort. Generally, the concept of tempo change itself implies change in birth timing when age changes with year, and not with cohort. Hence, by definition, the tempo change is not within the scope of indexing PTFR as it is traditionally conceptualized based on the idea of a synthetic cohort, and the consideration of the tempo effect in indexing PTFR is an attempt to incorporate an element of cohort fertility rate into the indexing of period fertility rate. We should be aware of this fact in understanding and interpreting "adjusted" indices.

The counterfactual interpretation of the B-F index is problematic because such an interpretation requires several strong assumptions, none of which hold in reality. These assumptions are (1) the uniform distribution of births within the "range of adjustment" for each age (the violation of which makes it impossible for the ratio of quantum to be expressed simply by 1-r(*t*)), (2) no change in the shape of the distribution from the previous year (the violation of which causes the observed age difference to be unequal to the underlying tempo change), (3) no change in the quantum of cohort fertility rates for all cohorts of women from year *t*-1 to year *t* (the violation of which makes it impossible to decompose the observed change in cohort fertility into the tempo and quantum components), and (4) no carryover effects of tempo change from year *t*-2 or before (the violation of which requires us to take into account, for example, the effect of postponed childbirths between year *t*-2 to year *t*-1 on childbirths between year *t*-1 and year *t*).

It seems that the only reasonable interpretation of "adjusted TFR" is the one that Yi and Land (2002) propose that it is "the average total number of births per women of a hypothetical cohort that has gone through the imagined extended period [of about 35 years] with changing tempo but constant quantum and invariant shape of the schedule" (page 270). In other words, this is an estimate of the quantum of fertility for a different synthetic cohort, and it will be equal to CFR if the given $r(t)$, as well as other conditions about the shape and quantum of the distribution, is fixed after time t into the future for about 35 years. In other words, the adjusted index is an index of prospective CFR, as an unadjusted TFR is, but under a distinct assumption about synthetic cohort.

While this interpretation is valid, there are still four issues for the use of the adjusted TFR. One issue is that if $r(t)$ is temporary unstable, it is problematic to use $r(t)$ estimated from the tempo change between year t-1 and t. Suppose that $g_p(t,x)$ shifted to older ages by the amount of $2r$ from year t-2 to year t. Then, If the amount of shift is $2r$-w from year t-2 to year t-1 and w from year t-1 to t, $TFR_{adj}(t)$, defined as TFR(t)/(1-r(t)), is TFR(t)/(1-w). However, if $r(t)$ is unstable, w can be very different from r, which is the mean change in age at childbirth during the past two years, and the use of one-year unit in measuring $r(t)$ yields meaningless fluctuations in the adjusted index. Since one year is an arbitrary time unit for measuring $r(t)$, we may instead use change in the mean age at birth in a fixed length of time, which will reduce fluctuations in the adjusted index.

The second issue is that the observed difference in the mean age at birth between year t-1 and t is equal to the amount of tempo change under a restrictive condition of no change in the shape of the distribution. If only the tempo and quantum of the distribution changes without changing its shape between times t-1 and t, such that

$g_p(t,x) = a(t)g_p(t-1, x-r(t))$, we obtain, $r(t) = E_{p,t}(x) - E_{p,t-1}(x)$, where $E_{p,t}(x)$ is the *conditional* mean of age at childbirth at year *t* given the condition that the childbirth occurs. However, if the shape of the distribution changes, no such simple relationship exists.

The third issue applies only to the B-F index. Since TFR_{CONV}, on which the B-F index is based, suffers from spurious tempo bias, as we have demonstrated, the B-F index also suffers from it. Hence, as an alternative to the B-F index, we propose the use of an adjusted index based on $\text{TFR}_{\text{SUV\_N}}(t)$. From equation (7), we can obtain

$$\text{Adjusted TFR}_{\text{SUV\_N}} = 1-(1-\text{TFR}_{\text{SUV\_N}}(t))^{\frac{1}{1-r(t)}} \qquad (16)$$

as the fertility quantum of an alternative synthetic cohort of women who are subject to a constantly shifting age distribution of births with the amount of tempo shift *r*(*t*).

The fourth issue also applies only to the B-F index. Since TFR_{CONV} can be regarded as being based implicitly on a Poisson-process model for multiple births at each age (Krishnamoothy 1979), and is, therefore, meaningful only as an index for the childbirth of all parities, it is problematic to adjust TFR_{CONV} for each parity as the B-F index does.

EMPIRICAL ESTIMATION METHOD $\text{TFR}_{\text{SUV\_N}}$ AND $\text{TFR}_{\text{SUV\_R}}$

In this section, we describe a statistical method of estimating $\text{TFR}_{\text{SUV\_N}}$ and $\text{TFR}_{\text{SUV\_R}}$ from empirical data based on the following assumptions: (1) fertility rates depend only on the combination of age and parity, and (2) each woman follows the schedule of parity-specific age-specific fertility hazard rates observed in a particular year without experiencing death during reproductive ages.

1. The Index of PTFR with Childbirths as Separate Nonrepeatable Events: $\text{TFR}_{\text{SUV\_N}}$

For TFR_{SUV_N}, let $B(i,x)$ be the number of childbirths of the *i*th order among 1,000 women aged *x* who bore *i*-1 *or fewer* children (including zero children) before becoming age *x*. We assume that $B(i,x)$ is given from the data of each year. Then $h(i,x)$ = $B(i,x)/1000$ is the hazard rate of bearing the *i*-th child at age *x*.

Let $S(i,x)$ be the survivor function of not having borne the *i*th child by the *end* of age *x* under the condition that the women enter the risk of having the *i*th child at the beginning of reproductive age x_0. By definition, $S(i, x_0 - 1) = 1$ for each *i*. By using the standard Kaplan-Meier method for estimating the survival probability (e.g., Miller 1981).

$$S(i,x) \equiv (1-h(i,x))S(i,x-1) = \prod_{k=x_0}^{x} (1-h(i,k)) \qquad (17)$$

$$\mathrm{TFR}_{\mathrm{SUV\_N}}(i) = 1 - S(i, x_{MAX}) \quad . \qquad (18)$$

2. The Index of PTFR with Childbirths as a Repeatable Event: TFR_{SUV_R}

For TFR_{SUV_R}, let $B(i,x)$ be the number of childbirths of the *i*-th order among 1,000 women aged *x* who bore exactly *i*-1 children before becoming age *x*. The hazard rate of having a child of the *i*th order is given as $h(i,x) = B(i,x)/1000$. The empirical index of TFR_{SUV_R} described below differs slightly from the index of PATFR described by Rallu and Toulemon (1994) and others. PATFR is based on the parity distribution at each age, $P(i|x)$, estimated by the following recursive equations for given $h(i,x)$.

$$P(0|x_0-1)=1; P(i|x_0-1)=0 \text{ for } i \geq 1, \qquad (19a)$$

$$P(0|x) = P(0|x-1)(1-h(1,x)) \text{ for } x \geq x_0, \qquad (19b)$$

$$P(i|x) = P(i-1|x-1)h(i,x) + P(i|x-1)(1-h(i+1,x)) \text{ for } i \geq 1, x \geq x_0. \qquad (19c)$$

The estimation of PTFR(*i*) for parity *i* given as $\sum_{j \geq i} P(j|x_{\mathrm{MAX}})$, however, implicitly assumes that no women bear more than one child at each age, and therefore, PATFR tends to underestimate the true PTFR. We therefore define an empirical index slightly differently based on conditional survival probabilities. The method of calculating TFR_{SUV_R} for the first childbirth is the same as that for TFR_{SUV_N} and PATFR.

For second or later births, each person enters the risk of having the childbirth not at the beginning of reproductive age x_0 but at some time $z \geq x_0$. The *conditional* survival probability that a woman who enters the risk of bearing a child of the *i*th order at the *beginning* of age *z* does not bear the *i*th child by the *end* of age *x*, $S(i,x|z)$, is given, under the assumption that only age dependence, and not duration dependence, of hazard rate exists, as

$$S(i,x|z) = \prod_{k=z}^{x}(1-h(i,k)) = S(i,x)/S(i,z-1) \quad . \tag{20}$$

Accordingly, the probability that such a woman bears a child of the *i*th order by the *end* of age *x* is $1 - S(i,x)/S(i,z\text{-}1)$.

Let $P(i,z)$ be the probability that a woman enters the risk of bearing a child of the *i*th order at the *beginning* of age *z*. Then the probability of bearing a child of the *i*th order by the *end* of age *x*, $Q(i,x)$, for a women of undetermined *z*, with $TFR_{SUV\_R}(i)$ being equal to this quantity at the maximum reproductive age x_{MAX}, is given as

$$\mathrm{Q}(i,x) \equiv \sum_{z=x_0}^{x} P(i,z)[1-S(i,x|z)] = \sum_{z=x_0}^{x} P(i,z)[1-S(i,x)/S(i,z-1)] \tag{21}$$

$$\mathrm{TFR}_{\mathrm{SUV\_R}}(i) = Q(i,x_{\mathrm{MAX}}) \quad . \tag{22}$$

Generally, equations (21) and (22) provide an index for PTFR for each given definition of $P(i,x)$. For example, if we define $P(i,x)$ to be equal to $Q(i-1,x-1)-Q(i-1,x-2)$, which means that the proportion of women who enter the risk of bearing the *i*th child at the beginning of age *x* is equal to the proportion of women who bore the (*i*-1)th child at age *x*-1, we obtain, though a proof is omitted, $TFR_{SUV\_R}(i)$ = PATFR(*i*), which is identical to the index derived from the recursive equations (19a,b,c). However, this model assumes that no women can bear more than one child at each age, and since this additional assumption is not made for the other two indices, TFR_{CONV} and TFR_{SUV_N}, we consider it to be undesirable for comparison. Hence, we assume instead that women enter the risk of having the *i*the child at the time of their previous birth and, therefore, $P(i,x)$ is approximated by the proportion of women who enter the risk of bearing the *i*th child from six months before to six months after the *beginning* of age *x*. In addition, we assume that the number of childbirths within six months can be approximated by the half of those within each age. It follows that

$$\begin{aligned} P(i,x) &= (1/2)[Q(i-1,x)-Q(i-1,x-1)]+(1/2)[Q(i-1,x-1)-Q(i-1,x-2)] \\ &= (1/2)[Q(i-1,x)-Q(i-1,x-2)]. \end{aligned} \quad (23)$$

By combining equations (21), (22), and (23), we can obtain $TFR_{SUV\_R}(i)$ successively for any given $i \geq 2$. Note that $TFR_{SUV\_R}(i)$ defined in this way is greater than PATFR(*i*).

3. A Note on the Measurement of $r(t)$ for Adjusted Indices

The tempo change $r(i,t)$ used for adjusted indices is equal to the difference in the mean age of childbearing by parity *i* for the distribution of ages at childbirth constructed from the period incidence rate under the assumption that there is no change in the shape of the distribution. Although the shape of the distribution may actually change, we employed this mean age difference to estimate $r(t)$ in calculating the two adjusted indices.

APPLICATION

1. Data

We analyze the recent trend in PTFR for Japan. The Japanese data from government publications do not give direct enumerations of the population at risk, that is, the number of women who have ever borne each given number of children at each age at each year. Hence, we estimated the proportion of women who are at risk of bearing the *i*th child by age *x* at year *t*, $R_i(t,x)$, from the cohort fertility data as follows:

$$R_i(t,x) = 1 - \sum_{u=15}^{x-1} n_{i-1,t-x,u} / N_{t-x,u} \qquad (24)$$

where $n_{i-1,t-x,u}$ is the number of the (*i*-1)th child born from the cohort of women with birth year *t-x* at age *u*, and $N_{t-x,u}$ is the population of women with cohort *t-x* at age *u*. It follows that the population at risk for TFR_{SUV_N} is estimated as $R_i(t,x)N_{t-x,x}$ and the population at risk for TFR_{SUV_R} is estimated as $(R_i(t,x) - R_{i-1}(t,x))N_{t-x,x}$. The data for $N_{t,x}$ are obtained from the Annual Report on Current Population Estimates that is published every year by adjusting numbers of the preceding Population Census with records from vital statistics and migration statistics.

For the data for the number of childbirths, Vital Statistics gives the number of live childbirths by parity and by age in 1965 and every year since 1968. For years 1947-1964, and 1966-1967, the data on the number of childbirths are available for (1) each age, but not by parity, and (2) by parity and by five-year age range. We employed the Stephen-Deming iterative proportional adjustment to estimate the number of childbirths by parity and by age, by adjusting the marginal frequencies of childbirths by age and the two-way marginal frequencies of childbirths by parity and by 5-year age categories to be the same as observed frequencies but by assuming the odd ratios of frequencies by parity and by single age within each five-year age category to be the same as those for the nearest year at which data for childbirths by parity and by age are available. However, data for parity-specific childbirths are consistently available for births of the first four children, while the numbers of the fifth or higher-order births are often aggregated, and, therefore, the components of TFR_{SUV_N} and TFR_{SUV_R} that require the numbers of parity-specific childbirths are calculated up to the fourth child. The estimate for TFR for the fifth and higher-order children for TFR_{SUV_N} and TFR_{SUV_R}, which has been very small (less than 0.01) in recent 20 years, is replaced by that of TFR_{CONV}, which does not require any distinction of birth order. Since data are available from year 1947 and those aged 15 in 1947 reach age 49 in year 1981, we calculated various TFR indices from year 1981 to 2001, given the assumption that reproductive ages are [15,49]. A very small number of births under age 15 are added to births at age 15, and those at age 50 or over are added to those at age 50 in order to cover all childbirths.

2. Analysis

Figure 3 presents TFR_{CONV}, TFR_{SUV}, adjusted TFR_{CONV}, and adjusted TFR_{SUV_N} for years 1981 to 2001 for the first child. The adjusted indices use the parity-specific change in mean age in the past two years to reduce some meaningless fluctuations of the graph based on change in the mean-age over one year.

(Figure 3 About Here),

The comparison of TFR_{CONV} and TFR_{SUV} in Figure 3 demonstrates the fact that although their difference has become somewhat smaller in recent years, there has been a considerable underestimation of PTFR by the use of TFR_{CONV}. The difference between TFR_{CONV} and TFR_{SUV} exceeded 0.1 around 1990. Figure 1 also shows that adjusted TFR_{CONV}, the B-F index, suffers from two problems. First, it is much more volatile than the adjusted TFR_{SUV}. Second, the adjustment can yield a misleading trend because, together with its volatility, it tends to inflate distortion caused by spurious tempo bias in TFR_{CONV}. Indeed, the graph indicates that the adjusted TFR_{CONV} gives a false impression that the quantum of the first childbirth under the assumption that the “current” tempo effect persists was increasing in early 1990s. On the other hand, the adjusted TFR_{SUV} under the assumption that the “current” tempo effect persists shows a stable declining trend. The trend shows that (1) throughout th past 20 years, spurious tempo bias (which is the difference between TFR_{CONV} and TFR_{SUV_N}) has been much greater than *hypothetical genuine tempo bias* which would be present if the "current" tempo change persisted thereafter (and which is measured by the difference between TFR_{SUV_N} and adjusted TFR_{SUV_N}), and (2) apparently because of a reduced change in the mean age at first childbirth in recent years, hypothetical genuine tempo bias has become very small, and therefore, the remaining bias in TFR_{CONV} in the past few years is largely spurious tempo bias which is affected by the accumulation of past tempo change.

Figure 4 presents TFR indices for the second child. Unlike Figure 3, this figure presents two survival probability indices of TFR, TFR_{SUV_N} and TFR_{SUV_R} , because they differ for second and later births.

(Figure 4 About Here)

While it is consistent across indices that the PTFR of the second child is declining steadily over time, Figure 4 again shows that TFR_{CONV} systematically underestimates PTFR because of spurious tempo bias and that adjusted TFR_{CONV} (the B-F index) yeilds a considerable distortion. While the two survival probability indices are fairly close, TFR_{SUV_R} consistently gives a lower value than TFR_{SUV_N}, though the difference has become small in recent years. A comparison of TFR_{CONV}, TFR_{SUV_N}, and adjusted TFR_{SUV_N} indicates that spurious tempo bias has been somewhat greater in size than hypothetical genuine tempo bias during the past 20 years, and hypothetical genuine tempo bias has diminished greatly in the past few years.

Figure 5 presents TFR indices for the third child, which also shows a steady decline throughout the 1990s. A conspicuous finding in this figure is that TFR_{CONV} and TFR_{SUV_N} are very close and so are adjusted TFR_{CONV} and adjusted TFR_{SUV_N}. This occurs because the TFR for the third child is much smaller than those of first and second children, and, therefore, as we have seen in Figure 1, spurious tempo bias becomes much smaller. Hence, the remaining tempo bias is largely hypothetical genuine tempo bias, whose amount has become steadily smaller since the late 1980s because of a steady reduction in change in the mean age at the third childbirth during that time. The level and the trend of TFR_{SUV_R} differ rather significantly from those of TFR_{SUV_N}. Since we do not know whether this indicates an improvement in measurement in TFR_{SUV_R} because of a finer control for people at risk, or bias in measurement because of neglect of duration dependence, this leaves some ambiguity regarding the level and trend of PTFR for the third child except for recent years, where a convergence between TFR_{SUV_N} and TFR_{SUV_R} is attained.

(Figure 5 About Here)

Figure 6 presents TFR indices for the childbirth of all parities, and Table 1 presents their numerical values. We see here a steady decline in PTFR since 1984 except for a small increase in 1994, while there was a slight trend toward an increase in PTFR during 1980-1984. Throughout these periods, TFR_{CONV} systematically underestimated PTFR because of large spurious tempo bias, and spurious tempo bias has been consistently greater than hypothetical genuine tempo bias. It is thus evident that by using TFR_{SUV_N} we can eliminate not only a large amount of bias in TFR_{CONV} but also a major portion of potential deviation of PTFR from the prospective CFR without making an additional assumption in measuring PTFR. The two survival probability indices show quite similar trends. Although the index was somewhat smaller for TFR_{SUV_R} than for TFR_{SUV_N} around 1990, there was a high congruence of the two in the early 1980s and late 1990s.

(Figure 6 and Table 1 About Here)

CONCLUSION

The major points we make in this article that there is spurious tempo bias in the conventional measure of TFR, that it is important to eliminate it, and that the elimination can be effectively accomplished by the use of $TFR_{SUV\_N.}$ Although this index was described formally by Keilman (1994), its usefulness because of its elimination of spurious tempo bias, and the significant weight of spurious tempo bias in the total tempo bias, has not been recognized before. Without making any additional assumption for PTFR other than that the underlying stochastic process of fertility is governed by the period hazard rate rather than governed by the period incidence rate, we can eliminate a major portion of the potential tempo bias, as we demonstrated both theoretically and empirically, and can thereby increase the association between PTFR and CFR.

While we therefore consider TFR_{SUV_N} to be useful as a general replacement for the conventional TFR, this replacement implies that government administrators of vital statistics and the population census should make the numbers of women by age and by parity publicly available so that they can be used for calculating TFR_{SUV_N} in comparing PTFR over time and among countries. Currently, however, such data are often not available. Because of the absence of such data, the present analysis also had to estimate the proportion of women who have not yet borne a child of a given parity at each age from the data for cohort fertility rates.

Second, we recommend the use of two other indices, TFR_{SUV_R} and adjusted TFR_{SUV_N}, for supplementary analyses, with some caveats in their use. TFR_{SUV_R}, which is a slight modification of the parity- and age-specific TFR (PAFTR) described by Rallu and Toulemon (1994), is useful to see whether a change in the definition of women by assuming the childbirth as a repeatable event, and the consequent control for a major aspect of population heterogeneity, leads to a different prediction. However, since its formal relationship to CFR is not clear, we cannot use this index to assess the tempo effect on period fertility rate. In addition, because of its neglect of duration dependence and the potential instability of its empirical estimate when the index is relaxed to reflect both duration and age dependence, TFR_{SUV_R} may not be as reliable as TFR_{SUV_N}.

Adjusted TFR_{SUV_N} provides an estimate for PTFR under an alternative assumption about the synthetic cohort, such that a group of women will follow the fertility schedule which is the same as that of the age-specific period hazard rates of a given year except that there is a constantly changing tempo of fertility. This index is clearly better than the index advocated by Bongaarts and Feeney (1998) for the same purpose because the former, and not the latter, is not affected by spurious tempo bias. However, there are two limitations in the use of adjusted TFR_{SUV_N}. First, substantively, if the rate of change in the tempo is unstable, the index will be of limited value. We have seen in the Japanese data that hypothetical genuine tempo bias, measured by the difference between TFR_{SUV_N} and adjusted TFR_{SUV_N}, is not temporally stable and has diminished greatly in recent years. If such instability in the tempo effect over time exists, there does not seem to be of great value in considering a prospective discrepancy between CFR and PTFR by assuming time-invariance in the tempo effect for many years into the future. Second, technically, we still see a problem for the empirical estimation of the time-varying tempo effect, $r(t)$, when the shape of the distribution of ages at birth changes over time.

For these reasons, we consider both TFR_{SUV_R} and adjusted TFR_{SUV_N} useful only for supplementary analyses. On the other hand, we consider the abolition of TFR_{CONV} and the adoption of TFR_{SUV_N} as the new standard measure of PTFR to be crucial in assessing the period fertility rate, because the weight of spurious tempo bias in the total tempo bias is large in the fertility of low parities, whose trend greatly affects the population projection, especially for countries with low fertility rates.

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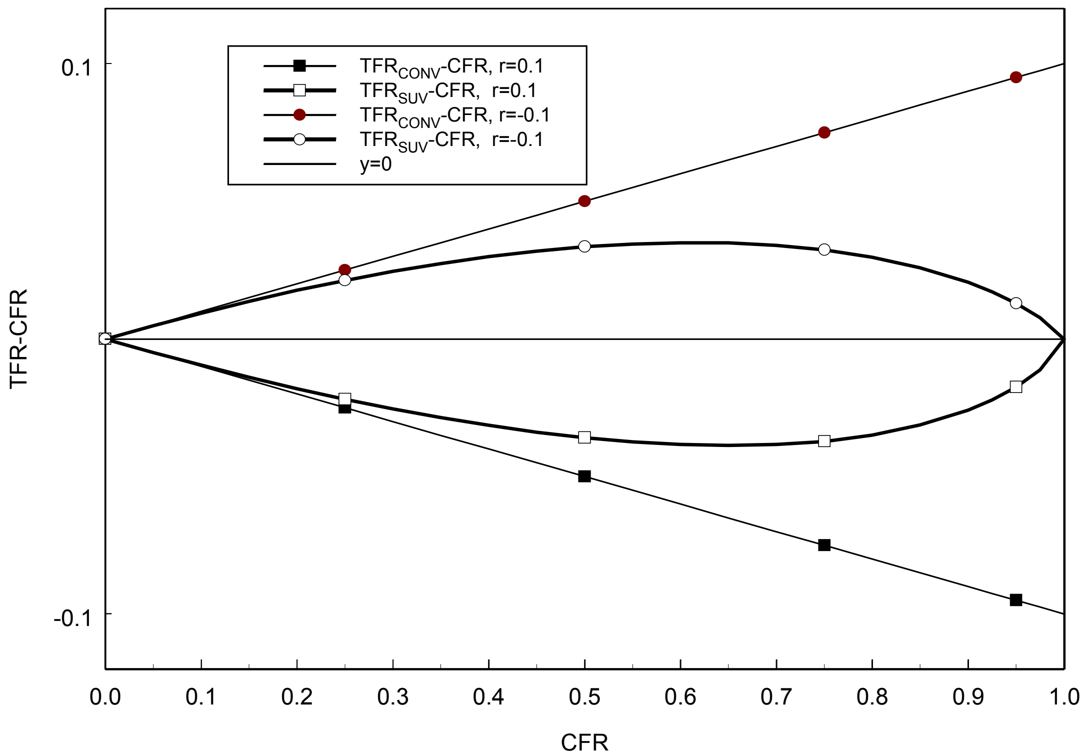
Table 1. TFR Indices by Year

Year	TFR_{CONV}	TFR_{SUV_N}	TFR_{SUV_R}	Adjusted TFR_{CONV}	Adjusted TFR_{SUV_N}
1981	1.741	1.884	1.888	1.903	1.961
1982	1.770	1.896	1.903	1.942	1.978
1983	1.801	1.911	1.924	1.966	1.994
1984	1.811	1.919	1.926	1.973	2.000
1985	1.764	1.898	1.880	1.932	1.978
1986	1.723	1.876	1.845	1.931	1.973
1987	1.691	1.855	1.811	1.924	1.969
1988	1.656	1.834	1.775	1.901	1.961
1989	1.572	1.777	1.702	1.817	1.916
1990	1.543	1.745	1.670	1.768	1.876
1991	1.535	1.717	1.664	1.727	1.832
1992	1.502	1.679	1.630	1.693	1.794
1993	1.458	1.632	1.585	1.668	1.756
1994	1.500	1.642	1.619	1.714	1.764
1995	1.422	1.575	1.537	1.589	1.676
1996	1.425	1.563	1.527	1.595	1.664
1997	1.388	1.521	1.486	1.546	1.620
1998	1.384	1.500	1.478	1.504	1.580
1999	1.342	1.452	1.435	1.437	1.519
2000	1.359	1.446	1.444	1.418	1.489
2001	1.334	1.413	1.413	1.368	1.438

Figure 1: Difference between TFR and CFR



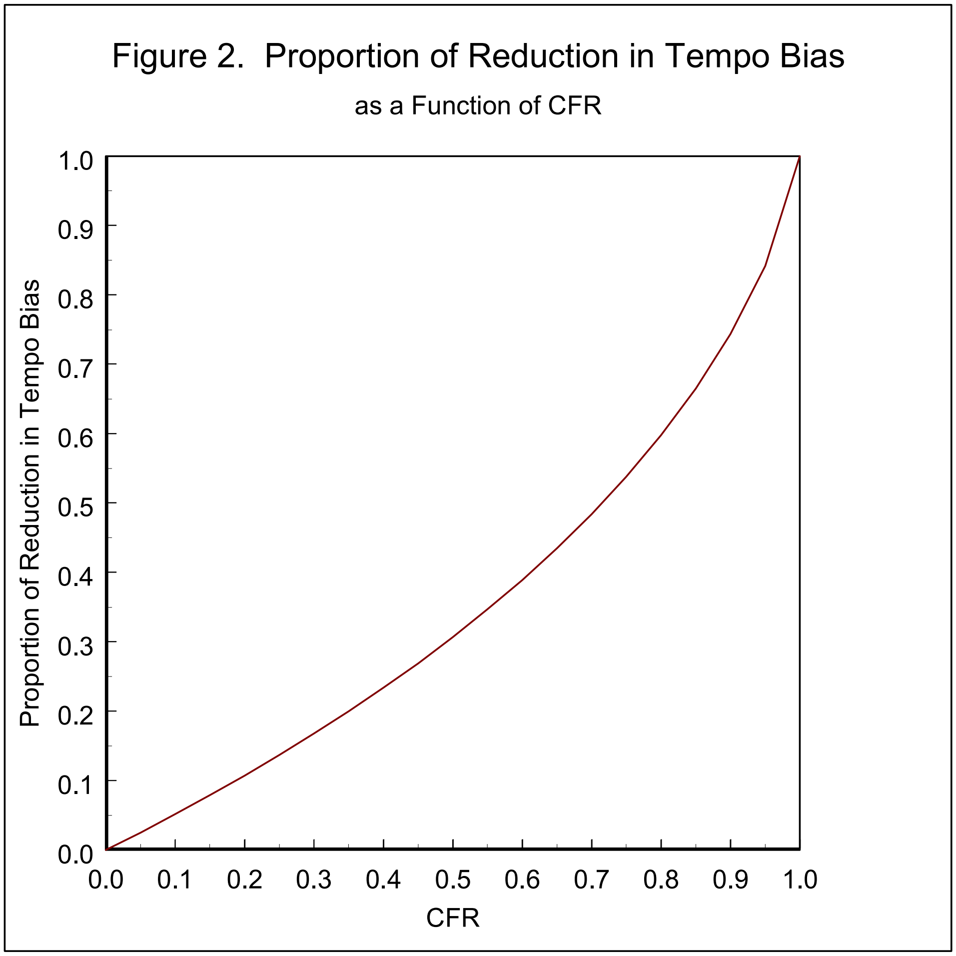


Figure 2. Proportion of Reduction in Tempo Bias
as a Function of CFR
Proportion of Reduction in Tempo Bias
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
CFR

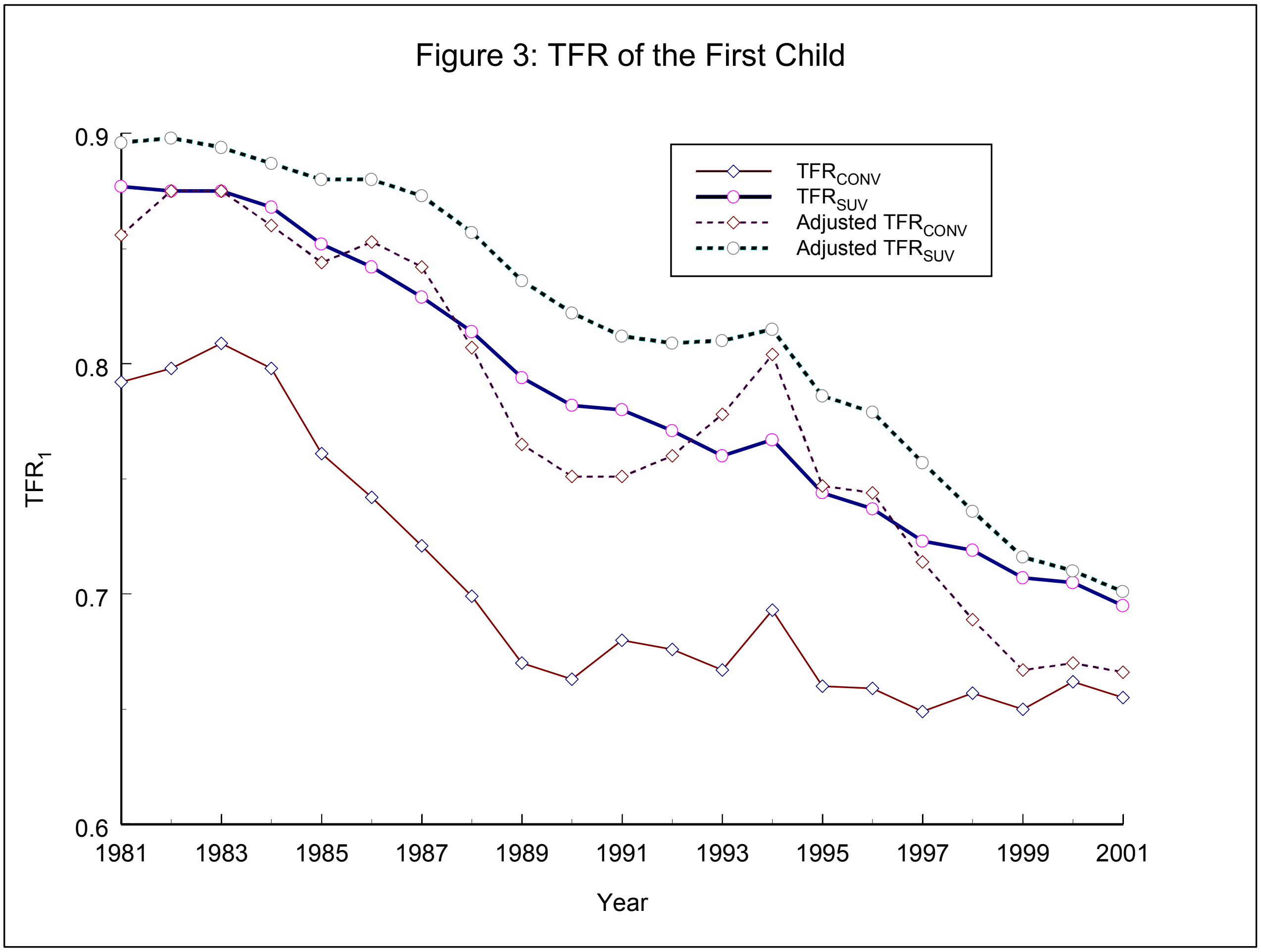


Figure 3: TFR of the First Child
TFR_CONV
TFR_SUV
Adjusted TFR_CONV
Adjusted TFR_SUV
0.9
0.8
0.7
0.6
TFR_1
1981
1983
1985
1987
1989
1991
1993
1995
1997
1999
2001
Year

Figure 4. TFR of the Second Child

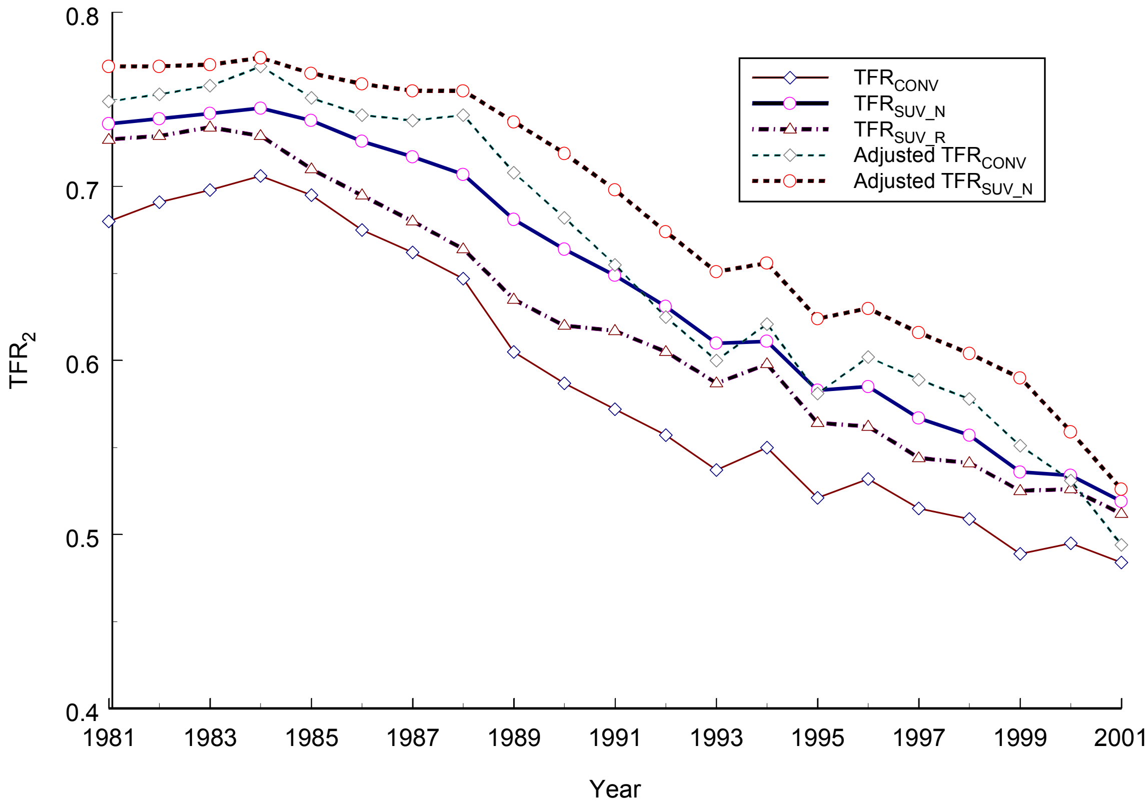
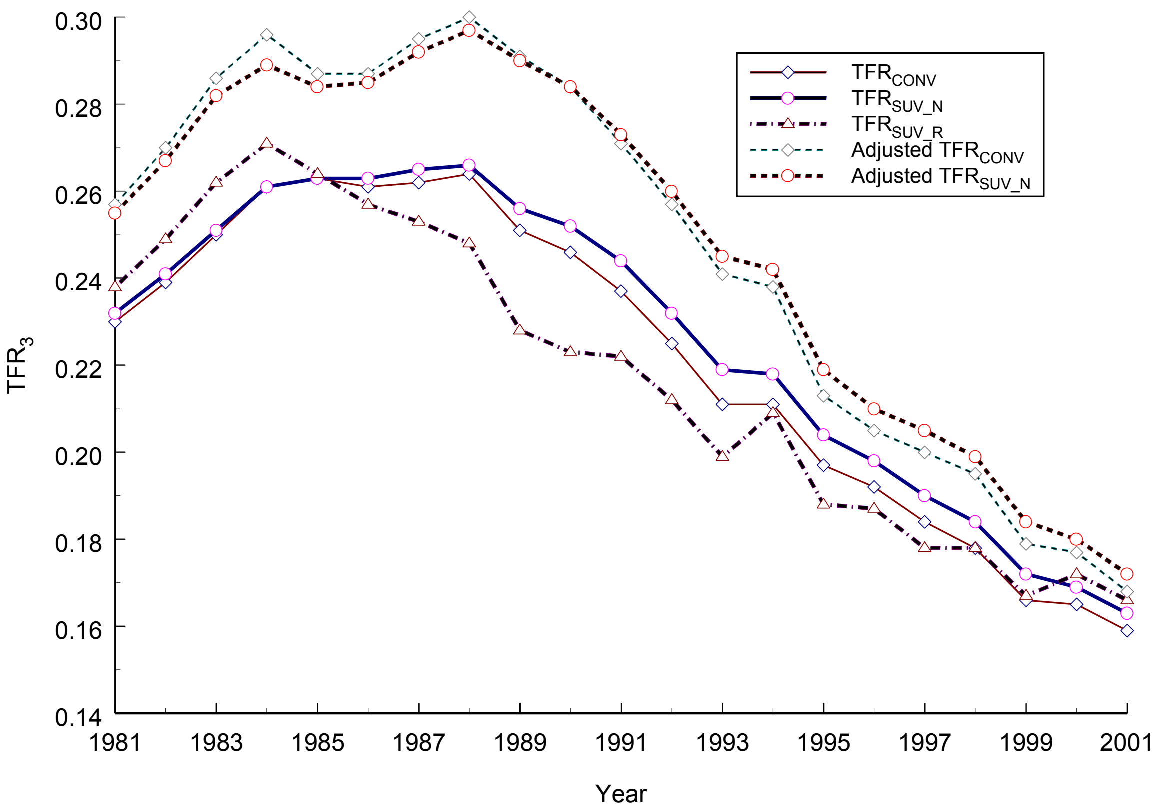


Figure 5. TFR of the Third Child



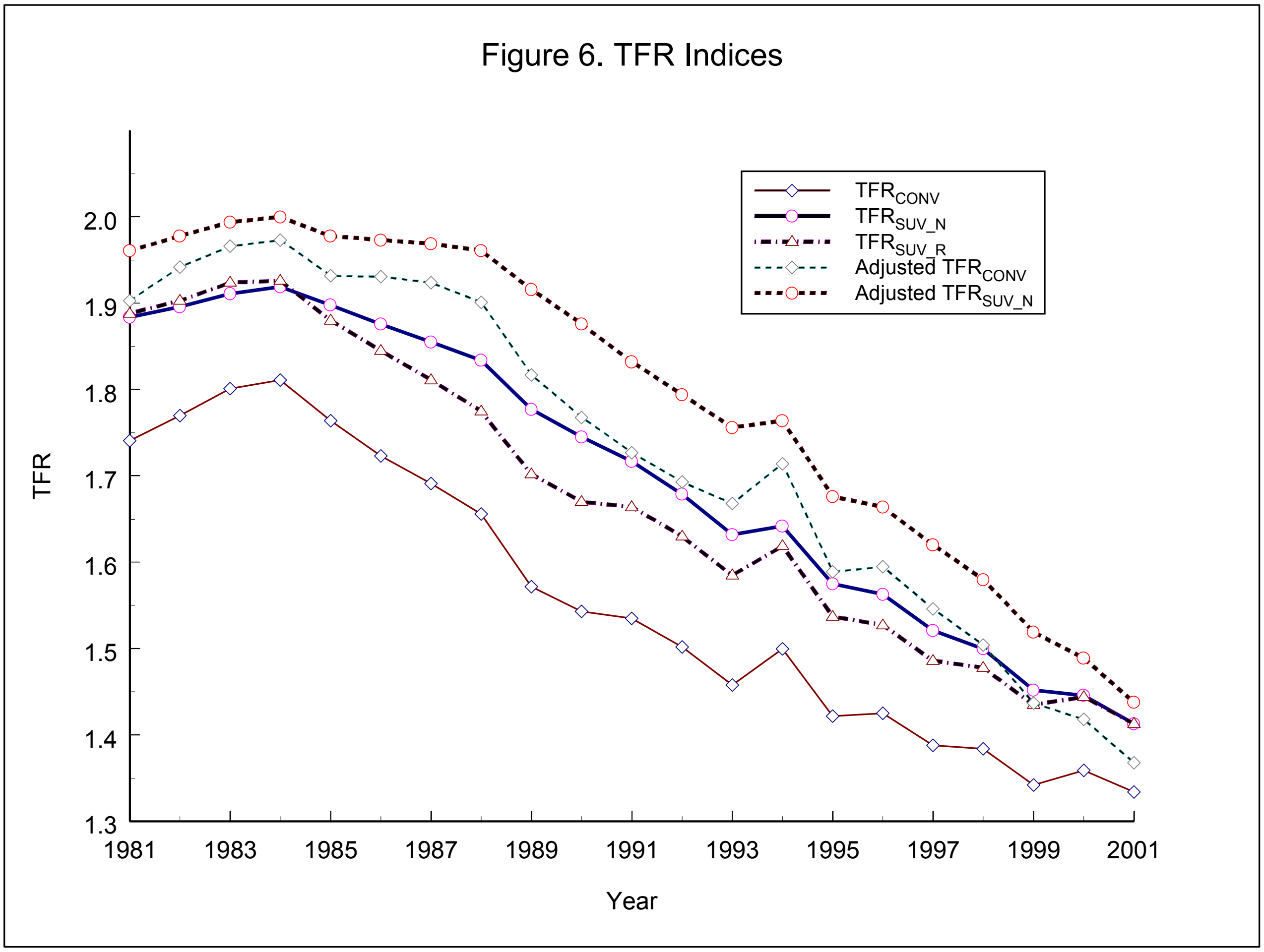


Figure 6. TFR Indices
TFR
Year
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1981
1983
1985
1987
1989
1991
1993
1995
1997
1999
2001
TFR_{CONV}
TFR_{SUV_N}
TFR_{SUV_R}
Adjusted TFR_{CONV}
Adjusted TFR_{SUV_N}